(KNOWLEDGE FOR DEVELOPMENT)

KIBABII UNIVERSITY

(KIBU)

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

YEAR ONE SEMESTER TWO

FOR THE DEGREE IN

(COMPUTER SCIENCE)

COURSE CODE: CSC 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 12 /07 /2021 **TIME: 9.00 A.M. – 11.00 A.M.**

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE [COMPULSORY] [30 MARKS]

a. Differentiate between computation and deduction reasoning. **[2 marks]**

b. Using suitable examples differentiate between Modus Ponens and Modus Tollens. **[4 marks]**

c. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, $C = \{x, y, z\}$ and let $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$ and $S = \{(b, x), (b, z), (c, y), (d, z)\}$. Find $R°S$. **[4 marks]**

d. Use the Euclidean algorithm to find the greatest common divisor of 46 and 21.hence or otherwise find integers s and t satisfying that gcd(46, 21) = s • 46 + t • 21. **[4 marks]**

e. Determine all integers x such that $x \equiv 2 \pmod{46}$ and $x \equiv 1 \pmod{21}$. **[4 marks]**

f. The number of bacteria, double every hour, then what will be the population of the bacteria after 10 hours? **[2 marks]**

g. Suppose E is an event in a sample space S with P (E) > 0. Define probability that an event A occurs once E has occurred or the conditional probability of A given E. **[2 marks]**

h. Let A and B be mutually exclusive events. Define both product and sum rule of A and B. **[2 marks]**

i. Find the coefficient of $x3y4$ in the binomial expansion of $(2x–2y)9$. **[2 marks]**

j. A pair of fair dice is thrown. Find the probability that the sum is 10 or greater if:

(i) 5 appears on the first dice; **[2 marks]**

(ii) 5 appears on at least one dice. **[2 marks]**

QUESTION TWO [20 MARKS]

a. Differentiate between a Graph and a Tree with an example in each case. **[4 marks]**

b. Suppose that a graph G (V, E) is such that V1 is of degree 1, V2 is of degree 2, V3 vertices of degree 3 and V5 vertices of degree 4. Can such a graph exist? **[3 marks]**

c. Given that the graph Kn has 21 edges.

i. Find the number of vertices that Kn is composed of. **[2 marks]**

ii. Determine the degree of each vertex. **[2 marks]**

iii. Calculate the sum of the degrees of all its vertices. **[2 marks]**

d. Alice, Alice's husband, their son, their daughter, and Alice's brother were involved in a murder. One of the five killed one of the other four. The following facts refer to the five people mentioned:

1. A man and a woman were together in a bar at the time of the murder.
2. The victim and the killer were together on a beach at the time of the murder.
3. One of Alice's two children was alone at the time of the murder.
4. Alice and her husband were not together at the time of the murder.
5. The victim's twin was not the killer.
6. The killer was younger than the victim.

Which one of the five was the victim? **[3 marks]**

QUESTION THREE [20 MARKS]

a. Differentiate between linear and non-linear recurrences. **[2 marks]**

b. Find a recurrence relation and initial conditions for 1,5, 17,53,161,485… **[2 marks]**

c. Consider the second-order homogeneous recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 2$, $a_1 = 7$,

- **i.** Find the next three terms of the sequence. **[2 marks]**
- **ii.** Find the general solution. **[2 marks]**
- **iii.** Find the unique solution with the given initial conditions. **[2 marks]**

d. Solve the following recurrence relations:

- **i.** $a_n + a_{n-1} - 6a_{n-2}=0$, where $a_0=1$ and $a_1=4$. **[5 marks]**
- **ii.** $f_n= 10f_{n-1} - 25f_{n-2}$, where $f_0=3$ and $f_1=17$. **[5 marks]**

QUESTION FOUR [20 MARKS]

a. Define the following terms:

- **i.** Relatively prime **[1 mark]**
- **ii.** Modular arithmetic **[1 mark]**

b. Given as a=365 and b=211 find **g (a, b) =s(a) + v(b)** **[5 marks]**

c. Find a positive integer ***(a)*** such that when ***(a)*** is divided by **3** it gives a remainder of **2**, when divided by **5** remainder is **4** and when divided by **7** remainder is **6**. **[5 marks]**

d. Find the least positive values of x such that

- **i.** $84x-38\equiv79 \pmod{15}$. **[3 marks]**
- **ii.** $78+x\equiv3 \pmod{5}$ **[2 marks]**
- **iii.** $89\equiv(x+3) \pmod{4}$ **[3 marks]**

QUESTION FIVE [20 MARKS]

a. Suppose a student is selected at random from 100 students where 30 are taking mathematics, 20 are taking chemistry, and 10 are taking mathematics and chemistry. Find the probability (p) that the student is taking mathematics or chemistry. **[2 marks]**

b. In a certain University, 25% of the students failed mathematics (M), 15% failed chemistry (C), and 10% failed both mathematics and chemistry. A student is selected at random.

(i) If he failed chemistry, find the probability that he also failed mathematics. **[2 marks]**

(ii) If he failed mathematics, find the probability that he also failed chemistry. **[2 marks]**

(iii) Find the probability that he failed mathematics or chemistry. **[2 marks]**

(iv) Find the probability that he failed neither mathematics nor chemistry **[2 marks]**

c. A random sample with replacement of size n = 2 is drawn from the set {1, 2, 3}, yielding the following 9-element equiprobable sample space: S = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

(i) Let X denote the sum of the two numbers. Find the distribution f of X, and find the expected value E(X). **[3 marks]**

(ii) Let Y denote the minimum of the two numbers. Find the distribution g of X, and find the expected value E(Y). **[3 marks]**

Prove that: $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$ **[4 marks]**