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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR
FIRST YEAR SECOND SEMESTER
MAIN EXAMINATION

**FOR THE DEGREE OF BACHELOR OF SCIENCE AND
BACHELOR OF EDUCATION**

COURSE CODE: MAA 123

COURSE TITLE: INTEGRAL CALCULUS

DATE: 12/7/2021

TIME: 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over

KIBABII UNIVERSITY EXAMINATIONS
2021/2022 SEMESTER TWO
MAA 123. INTEGRAL CALCULUS: TIME 2 HRS

Attempt question one and any other two questions

QUESTION ONE

- (a) Draw on the same axes the graph of $\sin x$ and $\cos x$ for $0 \leq x \leq 2\pi$. Use Integration to determine the area for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$ (30 mks)
(5 mks)
- (b) (i) Expand $\sin(A + B)$ (1 mk)
(ii) Show that $\sin 2A = 2 \sin A \cos A$ (1 mk)
(iii) Hence or otherwise find $\int \sin x \cos x dx$ (2 mks)
- (c) (i) Find the partial fractions of $\frac{\sin x}{1 - \sin^2 x}$ (3 mks)
(ii) Hence determine $\int \frac{x}{1 - x^2} dx$ (5 mks)
- (d) (i) By the method of change of variables evaluate $\int \frac{1}{\sqrt{a - 3(x+1)^2}} dx$ (6 mks)
(ii) Evaluate $\int \frac{dx}{x^2 - 2x + 5}$ (4 mks)
- (e) Integrate by parts the function $\int x \cos x dx$ (3 mks)

QUESTION TWO

- (a) (i) Find the circumference of a circle given by $x^2 + y^2 = r^2$ (20 mks)
(5 mks)
(ii) If $x = 5 \cos 60^\circ$ find the definite circumference (3 mks)
- (b) Find the length of the cycloid whose equations are given by
 $x = a(\theta + \cos \theta)$ and $y = a(1 - \sin \theta)$ for $0 \leq \theta \leq \pi$ (6 mks)
- (c) Find the volume generated when the ^{area} are between the graphs $y = x^3$ and $1 \leq y \leq 8$ is rotated about the y axis through four right angles. (6 mks)

QUESTION THREE

- (a) (i) Simplify $\cos 3x - \cos x$ as a product of $\sin \alpha \sin \beta$ (20 mks)
(3 mks)
(ii) $\int \sin 2x \sin x dx$ (2 mks)
(iii) $\int \cos 3x \sin x dx$ (2 mks)
- (b) Find the following definite integrals
- (i) $\int_{-2}^0 (2x + 1)(x + 1) dx$ (4 mks)
(ii) $\int_{-1}^1 \frac{x+3}{\sqrt{1+2x^2}} dx$ (3 mks)
(iii) $\int_{\pi/3}^{3\pi/4} \tan \theta \sec \theta d\theta$ (3 mks)
- (c) Evaluate $\int e^{-2x} \cos 2x dx$ (3 mks)

QUESTION FOUR**(20mks)**

(a) Show that $\int \frac{dx}{\sqrt{f^2 - g^2 x^2}} = \frac{1}{f} \sin^{-1} \left(\frac{gx}{f} \right) + c$. Hence find $\int \frac{2}{\sqrt{3-4x^2}} dx$

(7 mks)

(b) Find $\int \frac{5x+2}{(x-2)^2(x+1)} dx$

(5 mks)

(c) Prove that $\int x \sin^{-1} x dx = \frac{2x^2-1}{4} \sin^{-1} x + \frac{x(1-x^2)^{1/2}}{4} + c$

(8 mks)

QUESTION FIVE**(20mks)**

(a) Sketch the graphs

(i) $y = x^2 - 2x$

(2 mks)

(ii) $y = 2x - x^2$

(2 mks)

Show clearly the intercepts and the turning points. Hence or otherwise find the area enclosed by the two graphs.

(4 mks)

(iii) Determine the volume if the area is rotated through 360° about the x-axis

(4 mks)

(b) A curve $y = x^3 - x^2$ is revolved about the x-axis from $x = 1$ to $x = 2$ through two right angles.

(i) Give a sketch of the volume generated.

(3 mks)

(ii) Calculate this volume.

(5 mks)