(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAP 311**

COURSE TITLE: REAL ANALYSIS II

DATE: 12/7/2021 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30 MARKS)

a) Define the following:
   (i) Metric space. (4 marks)
   (ii) Limit. (2marks)
   (iii) Total Boundedness. (3marks)
   (iv) Sequential compactness. (2mark)
   (v) Relatively compact sets. (2 marks)

b) Suppose (X, d) is a metric space. Show that closed subsets of compact metric spaces are compact. (7marks)

c) Show that for every $x \in X$ and r >0 the open ball B(x, r) in a metric space is open. (10 marks).

QUESTION TWO (20 MARKS)

a) Define the following:
   (i) Continous function. (4 marks)
   (ii) Uniform continuity. (4 marks)

b) Show that for every nonempty set A ⊆ X the map X→ ℝ, x↦(x, A), is continuous. (6 marks)

c) Show that if $f \in (X, Y)$ and X is compact then the image f(X) is compact in Y.(6 marks).

QUESTION THREE (20 MARKS)

a) State the uniform continuity theorem (2 marks)

b) Define the terms:
   (i) Cauchy sequence. (4 marks)
   (ii) Complete metric space. (2 marks)
   (iii) Open cover and compactness. (5 marks)
   (iv) Open and closed set. (4 marks)

c) Show that a sequence in a metric space(X, d) has at most one limit. (10 marks).

QUESTION FOUR (20 MARKS)

a) Show that if U is a subset of a metric space (X, d), then $x \in U$ if and only if there exists a sequence (x_n) in U such that $x_n \to x$ and n → ∞ . (10 marks)

b) Show that if (x_n) is a sequence in a metric space (X, d) and $x_0 \in$ x, then that following statements are equivalent:
   (1) $\lim_{n \to \infty} x_n = x_0$
   (2) For every$\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that d $(x_n, x_0) < \varepsilon$ for all n ≥ n_0.

(10 marks)

QUESTION FIVE (20 MARKS)

a) Define the terms:
   (i) Open and closed ball. (4 marks)
   (ii) Accumulation points. (3 marks)
   (iii) Relatively compact sets. (1mark)
   (iv) Distance to a set. (2 marks)

b) Let (X, d) be a metric space.
   (i) Show that arbitrary union of open sets are open. (5 marks)
   (ii) Show that intersections of open sets are open. (5 marks)