(Knowledge for Development)

# KIBABII UNIVERSITY

## UNIVERSITY EXAMINATIONS

## 2020/2021 ACADEMIC YEAR

## FIRST YEAR SECOND SEMESTER

## MAIN EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

## (MATHEMATICS/PHYSICS/ CHEMISTRY)

**COURSE CODE:** MAA 121/MAT 102

**COURSE TITLE:** FOUNDATION MATHEMATICS II

**DATE:** 19/07/21 **TIME:** 2 PM -4 PM

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

## QUESTION ONE (30 MARKS)

(a) By reducing the given system of linear equations to echelon form, determine the values of $x_1$, $x_2$ and $x_3$ (5 mks)

$$3x_1 - 2x_2 - x_3 = 5$$
$$-2x_1 - x_3 + x_2 = -3$$

(b) Given $\boldsymbol{p} = -3i + 7j + 12k$ and $\boldsymbol{q} = 2j - 6k$ compute

(i) $-\boldsymbol{q} \times 2\boldsymbol{p}$ (3 mks)

(ii) $-\frac{1}{2}\boldsymbol{q}.\boldsymbol{p}$ (3 mks)

(c) Given that $I$ is an identity matrix find $M$ if $\begin{bmatrix} 5 & 15 \\ -1 & 2 \end{bmatrix} = (4I - M^T)^{-1}$ (5 mks)

(d) If $\vec{A}$ and $\vec{B}$ are vectors, prove that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ (5 mks)

(e) Find the inverse of the matrix using matrix inversion algorithm

$$\begin{bmatrix} 2 & -1 & 7 \\ 3 & 4 & -2 \\ 0 & 8 & 5 \end{bmatrix}$$ (9 mks)

## QUESTION TWO (20 MARKS)

(a) Determine if the two vectors are parallel, orthogonal or neither $6i + j - 5k$ and $2i - 3j + 2k$ (3 mks)

(b) If $detA = -10$ and $\det B = 2.6$ calculate $det(B^2A^3B^{-1}AB^T)$, given that matrices $A$ and $B$ are square matrices (4 mks)

(c) (i) Find $\emptyset$ so that $5\emptyset i + \emptyset j + 11k$ and $\emptyset i + j - 2k$ are perpendicular. (3 mks)

(ii) A plane is defined by 3 points P(0,0,1), Q(3,-1,2) and R(1,1,1), find a vector perpendicular to the plane. (5 mks)

(d) Compute the determinant of $\begin{bmatrix} 3 & -4 & 6 & 1 \\ -2 & -5 & 0 & 3 \\ 2 & -1 & 9 & 1 \\ 4 & 2 & -2 & 2 \end{bmatrix}$ (5 mks)

## QUESTION THREE (20 MARKS)

(a) If $A = \begin{bmatrix} a-x & -3x & p \\ b-y & -3y & q \\ c-z & -3z & r \end{bmatrix}$ Evaluate $detA$ given that

$$det\begin{bmatrix} a & p & x \\ b & q & y \\ c & r & z \end{bmatrix} = 15$$ (4 mks)

(b) Use Cramer's rule to find $x_1, x_2,$ and $x_3,$ (8 mks)

$$-3x + 4y + 6z = 23$$
$$x + 3y - 2z = -10$$
$$x + 3z = 6$$

(c) Compute the rank of

$$\begin{bmatrix} 3 & 5 & 6 & 2 \\ 3 & 8 & 5 & -3 \\ 1 & 5 & 2 & 1 \\ 2 & 6 & 0 & 9 \end{bmatrix}$$ (8 mks)

## QUESTION FOUR (20 MARKS)

(a) Given the matrix

$M = \begin{bmatrix} 1 & -5 & 10 \\ 2 & 0 & -3 \\ 3 & -1 & 6 \end{bmatrix}$, Compute $M(adjM)$ (10 mks)

(b) Find the solution of the following system of linear equations using Gaussian elimination with backward substitution (10mks)

$$2x_1 - 3x_2 + 2x_3 + x_4 = 4$$
$$x_1 + 2x_2 + 3x_3 + 4x_4 = 5$$
$$2x_1 - x_3 + 5x_4 = 4$$
$$5x_1 + 4x_2 + 3x_3 + 2x_4 = 1$$

## QUESTION FIVE (20 MARKS)

(a) Show that $\|\boldsymbol{a} \times \boldsymbol{b}\| = \|\boldsymbol{a}\|\|\boldsymbol{b}\| sin\theta$ (5 mks)

(b) Find the angle between the vectors $-2i - 4j + k$ and $3i + j - 5k$ (4 mks)

(c) Given that $A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & 3 & -6 \\ 1 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -10 & 9 \\ 1 & 12 & 3 \\ 0 & -1 & 2 \end{bmatrix}$

Prove that $\det(AB) = \det A \det B$ (5 mks)

(d) Solve the system by Gauss-Jordan elimination (6 mks)

$$x - 2y + 2z = 3$$
$$2x + y + z = 0$$
$$x + z = -2$$