(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **STA 313**

COURSE TITLE: STOCHASTIC PROCESSES I

DATE: 23/07/2021 **TIME**: 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1: (30 MARKS) (COMPULSORY)

a) Define the following terms as used in stochastic processes

i) Transient state **[1mk]**
ii) Cycling process **[1mk]**
iii) Jockeying **[1mk]**
iv) Reneging **[1mk]**

b) Let X have a Geometric distribution of the form

$$\boldsymbol{Prob\ (X = k) = p_k = q^{k-1}\, p\ , \quad k = 1,\ 2,\ 3, \ldots}$$

Obtain the probability generating function and hence find its mean and variance. **[6mks]**

c) A manufacturing company has a certain piece of equipment that is inspected at the end of each day and classified as just overhauled, good fair or inoperative. If the item is inoperative it is overhauled, a procedure that takes one day. Let us denote the four classifications as states 1, 2, 3 and 4 respectively. Assume that the working condition of the equipment follows a markov chain with the following transition matrix;

$$\begin{array}{c} \\ \\ \boldsymbol{P = Today} \end{array} \begin{array}{c} \\ \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \boldsymbol{Tomorrow} \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1/4 & 3/4 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

If it cost £ 225 to overhaul a machine (including lost time) on the average and £ 100 as productivity cost if a machine is found inoperative. Using the steady-state probabilities, compute the expected per day cost of maintenance. **[6mks]**

d) The arrival rate of customers at a banking counter follows poison distribution with a mean of 50 per hour. The service rate of the counter clerk also follows Poisson distribution with a mean of 65 per hour.

i) What is the probability of having 0 customers in the system (p_0)? **[1mk]**

ii) What is the probability of having 15 customers in the system (p_{15})? **[1mk]**

iii) What is the probability of having 20 customers in the system (p_{20})? **[1mk]**

iv) Find;

1) Average number of customers waiting in the system (in the queue and in the service station) (l_s) **[1mk]**
2) Average number of customers waiting in the queue (l_q) **[1mk]**
3) Average waiting time of customers in the system (in the queue and in the service station) $(\boldsymbol{w_s})$ **[1mk]**
4) Average waiting time of customers in the queue $(\boldsymbol{w_q})$ **[1mk]**

e) Cars arrive at a drive-in restaurant with a mean arrival rate of 21 cars per hour and the service rate of the cars is 15 per hour. The arrival rate and the service rate follow Poisson distribution .The number of parking space for cars are only 5. Find the standard results of this system. **[7mks]**

QUESTION 2: (20 Marks)

a) Let $\boldsymbol{X}$ have a Poisson distribution with parameter $\boldsymbol{\lambda}$ i.e.

$$\boldsymbol{Prob}\ (\boldsymbol{X} = \boldsymbol{k}) = \boldsymbol{p_k} = \frac{e^{-\lambda}\lambda^k}{k!}, \quad \boldsymbol{k} = \mathbf{0, 1, 2, 3, \ldots}$$

Obtain the probability generating function of X and hence obtain its mean and variance. **[6mks]**

b) On 1st January, 2017, Bakery A had 40% of its local market share while the other two Bakeries B and C had 40% and 20%, respectively, of the market share. Based upon a study by a marketing research firm, the following facts were compiled. Bakery A retains 90% of its customers while gaining 5% of B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and 7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's customers and 10 % of B's customers.

i. Formulate state transition matrix **[2mks]**

ii. Calculate the future probable market share **[4mks]**

iii. Determine the equilibrium conditions **[8mks]**

QUESTION 3: (20 Marks)

a) The school of international studies for population found out by its survey that the mobility of the population (in percent) of a state to a village, town and city is in the following percentages.

$$\begin{array}{cc} & \begin{array}{ccc} & To & \\ Village & Town & City \end{array} \\ From \begin{array}{r} Village \\ Town \\ City \end{array} & \begin{bmatrix} 50 & 30 & 20 \\ 10 & 70 & 20 \\ 10 & 40 & 50 \end{bmatrix} \end{array}$$

What will be the proportion of population in village, town and city after two years, given that the present population has proportions of 0.7, 0.2 and 0.1 in the village, town and city respectively? **[5mks]**

b) In a harbour, ships arrive with a mean rate of 20 per week. The harbour has 6 docks to handle unloading and loading of ships. The service rate of individual dock is 8 per week. The arrival rate and the service rate follow Poisson distribution. At a point in time, the maximum number of ships permitted in the harbour is 8.

Find;

i) Probability of having 0 ships in the system (p_0) **[3mks]**

ii) Average number of ships waiting in the queue (l_q) **[4mks]**

iii) Average number of ships waiting in the system (l_s) **[3mks]**

iv) Average waiting time of ships in the queue (w_q) **[3mks]**

v) Average waiting time of ships in the system (w_s) **[2mks]**

QUESTION 4: (20 Marks)

a) The number of units of an item that are withdrawn from inventory on a day-to-day basis is a markov chain process in which requirements for tomorrow depends on today's requirements. A one-day transition matrix is given below

Number of units withdrawn from inventory

$$\begin{array}{cc} & \textit{Tomorrow} \\ & \begin{array}{ccc} 5 & 10 & 15 \end{array} \\ \textit{Today} \begin{array}{c} 5 \\ 10 \\ 15 \end{array} & \begin{bmatrix} 0.6 & 0.4 & 0.0 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \end{array}$$

i) Construct a tree diagram showing inventory requirements on two consecutive days **[6mks]**

ii) Develop a two-day transition matrix **[2mks]**

iii) Comment on how a two day transition matrix might be helpful to a manager who is responsible for inventory management. **[2mks]**

b) In the machine shop of a small-scale industry, machines breakdown with a mean rate of 3 per hour. The maintenance shop of the industry has only one mechanic who can attend to the breakdown machines. The service rate of the mechanic is 2 machines per hour. Initially there are 6 working machines in the machine shop.

Find;

i) Probability of having 0 breakdown machines in the system (p_0) **[3mks]**

ii) Average number of breakdown machines waiting in the queue (l_q) **[2mks]**

iii) Average number of breakdown machines waiting in the system (l_s) **[2mks]**

iv) Average waiting time of customers in the queue (w_q) **[2mks]**

v) Average waiting time of customers in the system (w_s) **[1mk]**

QUESTION 5: (20 Marks)

a) Let $\boldsymbol{X}$ have a binomial distribution with parameter $\boldsymbol{n}$ and $\boldsymbol{p}$ i.e.

$$Prob\ (X = k\ ) = p_k = \binom{n}{k}\ p^k\ q^{n-k}\ , \qquad k = 0,1,2,3,\ldots,n$$

Obtain the probability generating function of $\boldsymbol{X}$ and hence find its mean and variance. **[7mks]**

b) Vehicles pass through a toll gate at the rate of 90 per hour. The average time to pass through the gate is 36 seconds. The arrival rate and service rate follow Poisson distribution. There is a complaint that the vehicles wait for long duration. The authorities are willing to install one more gate to reduce the average time to pass through the toll gate to 30 seconds if the idle time of the toll gate is less than 10% and the average queue length at the gate is more than 5 vehicles. Check whether the installation of the second gate is justified. **[5mks]**

c) At a central warehouse, vehicles arrive at the rate of 20 per hour and the arrival rate follows Poisson distribution. The unloading time of the vehicles follows exponential distribution and the unloading rate is five vehicles per hour. There are 5 unloading crews.

Find the following;

i) 1) Probability of having 0 vehicles in the warehouse **[3mks]**

2) Probability of having 4 vehicles in the warehouse **[1mk]**

ii) 1) Average number of vehicles waiting in the queue $(\boldsymbol{l_q})$ **[1mk]**

2) Average number of vehicles waiting in the system $(\boldsymbol{l_s})$ **[1mk]**

3) Average waiting time of vehicles in the queue $(\boldsymbol{w_q})$ **[1mk]**

4) Average waiting time of vehicles in the system $(\boldsymbol{w_s})$ **[1mk]**