(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 421

COURSE TITLE: PDE I

DATE: 21/07/2021 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Form a partial differential equation associated with the primitive $z = f(x^2 + y^2)$.

(4 marks)

b) Solve the Lagrange's Linear Equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. (7 marks)

c) By direct integration, solve $\dfrac{\partial^2 z}{\partial y^2} = z$; $y = 0$ then $z = e^x$, $\dfrac{\partial z}{\partial y} = e^{-x}$ (6 marks)

d) Solve the linear homogeneous partial differential equation

$\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$ (8 marks)

e) Solve the non-linear partial differential equation of the form $x^2p^2 + y^2q^2 = z^2$. (5 mks)

QUESTION TWO [20 MARKS]

(a) A Lagrange's Linear Partial differential equation is of the form

$Pp + Qq = R$, where P, Q and R are functions of x, y, z and $p = \dfrac{\partial z}{\partial x}, q = \dfrac{\partial z}{\partial y}$. Show that

its solution is given by $f(u, v) = 0$. (12 marks)

(b) Using the method of Multipliers, find the complete solution of the differential equation

$x(z^2 - y^2)\dfrac{\partial z}{\partial x} + y(x^2 - z^2)\dfrac{\partial z}{\partial y} = z(y^2 - x^2)$ (8 marks)

QUESTION THREE [20 MARKS]

(a) Find a complete integral of the partial differential equation $p(q^2 + 1) + (b - z)q = 0$

by Charpit's method. (10 marks)

(b) Find the complete solution for the equation $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$. (10 marks)

QUESTION FOUR [20 MARKS]

(a) Classify the partial differential equation $\frac{\partial^2 z}{\partial x^2} + x\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} = 0$. (6 marks)

(b) Using the method of separation of variables, solve;

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, when $u(x, 0) = 6e^{-3x}$. (14 marks)

QUESTION FIVE [20 MARKS]

(a) Obtain solution for the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such that $y = P_0 \cos pt$, (P_0 is a constant) when $x = l$ and $y = 0$ when $x = 0$. (14 marks)

(b) Solve $(w + y + z)\frac{\partial w}{\partial x} + (w + x + z)\frac{\partial w}{\partial y} + (w + x + y)\frac{\partial w}{\partial z} = x + y + z$ (6 marks)