(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS 2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: MAT 427

COURSE TITLE: NUMERICAL ANALYSIS III

DATE: 15/7/2021 **TIME:** 9 AM – 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) Given the Initial Value Problem $u' = t^2 + u^2, u(0) = 0$. Determine the first 3 non- zero terms in the Taylor series for $u(t)$ and hence get the value of $u(1)$. Also determine when the error in $u(t)$ obtained from the first two non-zero terms is to be less than 10^{-6} after rounding off. (8marks)

(b) Solve $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary conditions as shown in the figure below and their corresponding u_n^1 (8 marks)

0	11.1	17	19.7	18.6
0	u_1	u_2	u_3	21.9
0	u_4	u_5	u_6	21
0	u_7	u_8	u_9	17
0	8.7	12.1	12.8	9

(c) Evaluate the integral using series expansion method (4marks)

$$I = \int_0^1 \frac{e^x}{\sqrt{x}}\,\mathrm{dx}$$

(d) Evaluate the intergral $I = \int_0^1 \frac{dx}{1+x}$ using the Gauss-legendre two –point formulae. (4marks)

(e) Given the Boundary Value Problem $y'' = y'$; $y(0) = 1,\ y(1) = 2$. Solve (6 marks)

QUESTION TWO

Solve the heat equation

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ Subject to the initial and boundary conditions

$u(x, 0) = e^{-x^2 t} sin\pi\, x, 0 \leq x \leq 1$

$u(0, t) = u(1, t) = 0$

Using the following methods

(i) The Schmidt method (6marks)

(ii) The laasonen method (7marks)

(iii) The Crank-Nicklson method (7marks)

For $h = \frac{1}{3}$ and $k = \frac{1}{36}$, Intergrate upto two time levels.

QUESTION THREE

Solve the initial value problem $\dot{u} = 2tu^2,\ u(0) = 1$ withb $h = 0.2$ over the integral [0,1]. Use the forth order classical Runge –Kutta method. (20marks)

QUESTION FOUR

(a) Evaluate the integral of the following function
$y = \int_0^{\pi} sinx\, dx,\ 0 \leq x \leq \pi, h = \frac{\pi}{6}$

i. Simpson's $\frac{1}{3}$ *and* $\frac{3}{8}$ rule (4marks)

(b) Evaluate $I = \int_1^2 \int_1^2 \frac{dxdy}{x+y}$

Using trapezoidal rule with

i. $h = k = 0.5$ (6marks)

ii. $h = k = 0.25$ (10marks)

and modify the results using Romberg formulae

QUESTION FIVE

Solve the initial value problem
$\dot{u} = -2tu^2,\ u(0) = 1$, with $h = 0.2$ over the interval [0,1]
Using

i. Forward Euler Method (6marks)

ii. Backward Euler Method (7marks)

iii. Midpoint Euler Method (7marks)