(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAA 315

COURSE TITLE: ANALYTIC APPLIED MATHEMATICS 1

DATE: 13/7/2021 **TIME:** 9:00 - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) Given the ordinary differential equation (ODE)

$$3x\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=0$$

i). Find the singular point for the ordinary differential equation. Hence show that the ODE **can** be solved using Frobenius method. **(5 marks)**

ii). Using the Frobenius method, obtain the *indicial equation* for the ODE **(4 marks)**

iii). Determine the *recursion relation* for the ODE **(3 marks)**

iv). Develop a power series solution for the ODE **(3 marks)**

(b) Find the Laplace transform of the function $f(t)=e^{at}$, where a is a constant. **(5 marks)**

(c). Determine the Legendre's polynomial P_1 using Rodrigues formula. **(4 marks)**

(d). given the partial differential equation, $z=5x^4+2x^3y^2-3y$, evaluate $\frac{\partial^2 z}{\partial x^2}$ and $\frac{\partial z}{\partial y}$ when $x=2$ **(4 marks)**

(e). Show that the function $f(x)=x-x^3$ in the interval $-\pi\leq x\leq\pi$ is an even function **(3 marks)**

QUESTION TWO (20 MARKS)

(a) Use Laplace transforms to solve the second order ordinary differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0,\quad y(0)=2,\quad y'(0)=2$$

(7 marks)

(b) The gamma function is given by the integral

$$\Gamma(x)=\int_0^{\infty}(t^{x-1}e^{-t})dt,\ \ x>0.$$

Show that $\Gamma(x+1)=x\Gamma(x)$ **(4 marks)**

(c) Find the constant a_0 of the Fourier series for the function $f(x)=e^x$ in $-\pi<x<\pi$ **(3 marks)**

(d) If $z=(\frac{x}{y})ln\,y$,

(i). Show that $\frac{\partial z}{\partial y}=x\frac{\partial^2 z}{\partial y\partial x}$ **(3 marks)**

(ii). Evaluate $\frac{\partial^2 z}{\partial y^2}$ when $x=3$ and $y=1$ **(3 marks)**