(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR DEGREES OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

COURSE CODE: STA 225

COURSE TITLE: INTRODUCTION TO STATISTICS AND PROBABILITY

DATE: 4/10/2021 **TIME:** 9:00 AM - 11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Define the following research terms:

i. Universal set

ii. Mutually exclusive events

iii. Conditional probability

iv. Random variable **(8 marks)**

b. In a survey of 200 people that had just returned from a trip to Europe, the following information was gathered.

- 142 visited England
- 95 visited Italy
- 65 visited Germany
- 70 visited both England and Italy
- 50 visited both England and Germany
- 30 visited both Italy and Germany
- 20 visited all three of these countries

i. How many went to England but not Italy or Germany?

ii. How many went to exactly one of these three countries?

iii. How many went to none of these three countries? **(10 marks)**

c. If we twice flip a balanced coin, what is the probability of getting at least one head? **(4 marks)**

d. A die is rolled. If the die shows a 1 or a 6, a coin is tossed. What is the sample space for this experiment (first, represent the outcomes using a tree diagram)? **(4 marks)**

e. In a large metropolitan area, the probabilities are 0.86, 0.35, and 0.29, respectively, that a family (randomly chosen for a sample survey) owns a color television set, a HDTV set, or both kinds of sets. What is the probability that a family owns either or both kinds of sets? **(4 marks)**

QUESTION TWO (20 MARKS)

a. i) If A, B and C are any three events in a sample space S, then show that:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

(8 marks)

ii) If a person visits his dentist, suppose that the probability that he will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24, the probability that he will have a tooth extracted is 0.21, the probability that he will have his teeth cleaned and a cavity filled is 0.08, the probability that he will have his teeth cleaned and a tooth extracted is 0.11, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist will have NONE of these things done to him? **(5 marks)**

b. During an epidemic in a town, 40% of its inhabitants became sick. Of any 100 sick persons, 10 will need to be admitted to an emergency ward. What is the probability that a randomly chosen person from this town will be admitted to an emergency ward? **(4 marks)**

c. A fruit basket contains 25 apples and oranges, of which 20 are apples. If two fruits are randomly picked in sequence, what is the probability that both the fruits are apples? **(3 marks)**

QUESTION THREE (20 MARKS)

a) Suppose E denotes mathematical expectation, X is a random variable and $\mu = E(X)$, show that;

$$E(X-\mu)^2 = E(X^2) - \mu^2 = \text{Variance (X)}$$ **(5 marks)**

b) To find out the prevalence of smallpox vaccine use, a researcher inquired into the number of times a randomly selected 200 people aged 16 and over in an African village had been vaccinated. He obtained the following figures: never, 17 people; once, 30; twice, 58; three times, 51; four times, 38; five times, 7. Assuming these proportions continue to hold exhaustively for the population of that village, what is the expected number of times those people in the village had been vaccinated, and what is the standard deviation?. **(5 marks)**

c) Let Y be a random variable with pdf:

$$f(y) = \begin{cases} \frac{3}{64} y^2(4-y), & 0 \leq y \leq 4 \\ 0, & elswhere \end{cases}$$

a) Find the expected value and variance of Y

b) Let $X = 300Y + 50$. Find $E(X)$ and $var(X)$

c) Find $P(X > 750)$ **(10 marks)**

QUESTION FIVE (20 MARKS)

a) In the tossing of three fair coins, let the random variable X be defined as X = number of tails. Then X can assume the values 1, 2, and 3. Find the probabilities associated with the values of X. **(5 marks)**

b) Suppose that a large grocery store has shelf space for 150 cartons of fruit drink that are delivered on a particular day of each week. The weekly sale for fruit drink shows that the demand increases steadily up to 100 cartons and then levels off between 100 and 150 cartons. Let Y denote the weekly demand in hundreds of cartons. It is known that the pdf of Y can be approximated by:

$$f(y) = \begin{cases} y, & 0 \leq y \leq 1 \\ 1, & 1 < y \leq 1.5 \\ 0, & elswhere \end{cases}$$

a) Find $F(y)$

b) Find $P(0 \leq Y \leq 0.5)$

c) Find $P(0.5 \leq Y \leq 1.2)$ **(10 marks)**

c) Suppose that a fair coin is tossed twice so that the sample space is $S = \{HH, HT, TH, TT\}$. Let X be number of heads.

i. Find the probability function for X.

ii. Find the cumulative distribution function of X. **(5 marks)**