KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAA 225

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 5/10/2021 **TIME:** 2:00 PM – 4:00 PM

INSTRUCTIONS TO CANDIDATES

Answer question ONE and ANY **OTHER TWO (2)** questions

QUESTION ONE (30 MARKS)

a. State and prove the Residue Theorem (5 marks)

b. Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$ along the parabola $x = 2t, y = t^2 + 3$ (5 marks)

c. Given $f(z) = u + iv$ is analytic in a region $\mathbb{R}$. Prove that u and v are harmonic in $\mathbb{R}$ if they have continuous second partial derivatives in $\mathbb{R}$. (5 marks)

d. Let $f(z) = \ln(1 + z)$, where we consider the branch that has the zero value when $z = 0$, expand $f(z)$ in a Taylor series about $z = 0$. Hence expand $\ln(1 + z/1 - z)$ in a Taylor series about $z = 0$ (6marks)

e. Given that $f(z) = u + iv$ is an analytic function and suppose $u(x, y) = e^x(x \sin y - y \cos y)$, find $v(x, y)$ (9 marks)

QUESTION TWO (20 MARKS)

a. Find the residues of $f(z) = \dfrac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane and hence evaluate $\oint_c f(z)dz$ (10 marks)

b. Using Cauchy's integral formula, evaluate $\int_c \dfrac{2z^2 + z}{z^2 - 1} dz$ where C is $|z - 1| = 1$ (5 marks)

c. Determine if the function $U(x, y) = e^x(y\cos 2y + x \sin 2y)$ is harmonic. (5 marks)

QUESTION THREE (20 MARKS)

a. Show that $f'(z)$ for $f(z) = \bar{z}$ does not exist (5 marks)

b. Verify the Cauchy- Riemann equations for $f(z) = e^z$ (4 marks)

c. Given that $f(z) = u + iv$ is an analytic function and suppose that $u = x^2 + 4x - y^2 + 2y$, use the Cauchy-Riemann equations to determine the imaginary part v (6marks)

d. Show that if $f(z)$ is analytic and $f'(z)$ is continuous at every point inside and on a simple closed curve C then $\int_C f(z)dz = 0$ (5marks)

QUESTION FOUR (20 MARKS)

a) Evaluate $\int_{1+i}^{2+3i} (z^2 + z)dz$ along the line joining the points (1, -1) and (2, 3) (5marks)

b) If $f(z)$ is analytic within and on simple closed curve C and if a is any point within C, show that $f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)}{z-a}dz$ (5marks)

c) Prove that $\lim_{z \to i} \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i} = 4 + 4i$ (5 marks)

d) Using the definition of derivative, find the derivative of $w = f(z) = z^2 - 2z + 1$ at $z = z_o$ and $z = -1$ (5 marks)

QUESTION FIVE

a) Evaluate $\int \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$

(7 marks)

b) State the De Moivre's theorem and use it to solve $Z^6 = 3 - 4i$ (5 marks)

c) Evaluate $(-1 + 2i)^{1/3}$ and represent first three solutions graphically (8 marks)