(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

COURSE CODE: **MAP 324**

COURSE TITLE: **GROUP THEORY**

DATE: 01/10/21 **TIME: 2 PM - 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question ONE and Any TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

a. Define the following

   i. Proper subgroup (2marks)

   ii. Trivial subgroups (2marks)

   iii. Permutation (2marks)

   iv. Symmetric group (2marks)

b. Let G be a group. Show that the left cancellation holds i.e. for $x,y,z \in G$, $z.x = z.y \Rightarrow x=y$ (6marks)

c. Show that there is exactly one element $x^{/}$ such that $x.x^{/} = x^{/}.x = e$ (4marks)

d. Let G be a group and $a,b \in G$. Show that $(a.b)^{-1} = b$ (6marks)

e. Let G be a group and $a,b \in G$. Show that the equation $x.a = b$ has a unique solution (4marks)

QUESTION TWO (20MARKS)

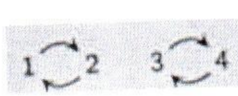
a. Define the following

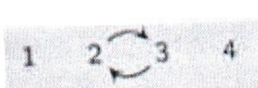
   i. Group (3marks)

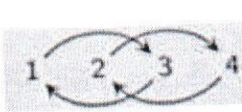
   ii. Subgroup (2marks)

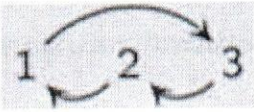
   iii. Alternating group (2marks)

b. Decompose the following permutations of s4 into disjoint cycles (4marks)

   i. 

   ii. 

   iii. 

iv. 

c. Compose the permutation (1234)*(13)(24) in cycle notations (4marks)

d. Let H be the subgroup of Z6 consisting of elements 0 and 3. Write down the cosets of H

(4marks)

QUESTION THREE (20MARKS)

a. Define the following

i. Normal subgroup (2marks)

ii. Index of a subgroup (2marks)

iii. Quotient group (2marks)

b. Let G be a group and N be a subgroup of G. Show that the following statements are equivalent

1. The subgroup N is normal in G
2. For all $g \in G$, $gNg-1 \subset N$ (4marks)

c. Let H be a subgroup of a group G. Show that the left cosets of H in G partition G.

(7marks)

QUESTION FOUR (20MARKS)

a. Define the following

   i. Center of a group (2marks)

   ii. Homomorphism (2marks)

   iii. Isomorphic (2marks)

b. Show that the center Z of a group G is a normal subgroup of G. (10marks)

c. Let $\varphi: G \to H$ be a homomorphism and let e,e$'$ denote the identity elements of G and H respectively. Show that

   1. $\varphi(e) = e'$
   2. $\varphi(a^{-1}) = \varphi(a)^{-1}$
   3. $\varphi(a^n) = \varphi(a)^n$ for all a∈G, n∈Z (4marks)

QUESTION FIVE (20MARKS)

a. Define the following

   i. Group action (2marks)

   ii. Transitive action (2marks)

   iii. The orbit (2marks)

b. Show that the orbits of an action partition X (5marks)

c. Show that stab(x) is a subgroup of G for each x∈X. (5marks)

d. Show that the cardinality of the orbit containing x∈X under the action of G is the index of the stab(x) (4marks)