



(KNOWLEDGE FOR DEVELOPMENT)

**KIBABII UNIVERSITY
(KIBU)**

**UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR**

**SPECIAL/SUPPLEMENTARY EXAMINATIONS
FIRST YEAR SEMESTER TWO**

**FOR THE DEGREE IN
(COMPUTER SCIENCE)**

COURSE CODE: CSC 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 30/09 / 2021 TIME: 02.00 P.M – 04.00 P.M

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO

QUESTION ONE [COMPULSORY] 30 MARKS]

- a. Studying logic means study proof. Justify this claim. [2 marks]
- b. When is it appropriate to apply deduction process and not computation process? At what point to these two strategies converge? [4 marks]
- c. Using relevant statements and formalization differentiate between Modus Ponens and Modus Tollens. [4 marks]
- d. Use the Euclidean algorithm to find the greatest common divisor of 46 and 21. Hence or otherwise find integers s and t satisfying that $\gcd(46, 21) = s \cdot 46 + t \cdot 21 = 11$ [4 marks]
- e. Determine all integers x such that $x \equiv 2 \pmod{46}$ and $x \equiv 1 \pmod{21}$. [3 marks]
- f. Write the following statement in symbolic form using quantifiers:
- (i) All students have taken a course in mathematics [2 marks]
- (ii) Some students are intelligent, but not hardworking [2 marks]
- g. What is the probability that when two dice are rolled, the sum of the numbers on the two dice is more than 8? [3 marks]
- h. Find the coefficient of x^3y^4 in the binomial expansion of $(2x-3y)^6$. [3 marks]
- i. Consider the sequence given recursively by $a_1=4$, $a_2=6$ and $a_n=a_{n-1} + a_{n-2}$. Write out the first 6 terms of the sequence. [3 marks]

QUESTION TWO [20 MARKS]

- a. Find the recurrence relation with initial condition for the following: 2, 10, 50, 250, ... [2 marks]
- b. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for $n=2, 3, 4, \dots$, and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3 ? [2 marks]
- c. Determine the generating function of the following numeric function
 $a_n = 2^n$, if n is even [2 marks]
- d. Use mathematical induction to show that $2 + 4 + 6 + \dots + 2n = n^2 + n$, for $n \geq 1$ [4 marks]
- e. Given two integers a and b such that $a=2740$ and $b=1760$ using extended Euclidian algorithm find the values of integers s and t such that $s(a)+t(b)=\text{Gcd}(a,b)$ [4 marks]

- f. Find a positive integer (x) such that when (x) is divided by 5 it gives a remainder of 4, when divided by 7 remainder is 6 and when divided by 9 remainder is 8. [6 marks]

QUESTION THREE [20 MARKS]

- a. Define the following terms as used in the study of discrete structures
- i. Linear Congruence [1 mark]
 - ii. Equiprobable Spaces [1 mark]
 - iii. Random Variables [1 mark]
 - iv. Independent Event [1 mark]
- b. A pair of dice is loaded. The probability that a 4 appears on the first die is $\frac{2}{7}$, and the probability that a 3 appears on the second die is $\frac{2}{7}$. Other outcomes for each die appear with probability $\frac{1}{7}$. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled? [3 marks]
- c. What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin containing 50 balls labeled with numbers 1, 2, ..., 50 if:
- i. The ball selected is not returned to the bin before the next ball is selected and; [3 marks]
 - ii. The ball selected is returned to the bin before the next ball is selected? [3 marks]
- d. There are many gambling games nowadays that award enormous prizes to people who correctly choose a set of six numbers out of the first n positive integers where n is usually between 30 and 60. What is the probability that a person picks the correct six numbers out of 40? [3 marks]
- e. Prove using counting argument that $C(n, r) = C(n-1, r) + C(n-1, r-1)$. [4 marks]

QUESTION FOUR [20 MARKS]

- a. Differentiate between a Graph and a Tree and a spanning tree with an example in each case. [4 marks]
- b. Suppose that a graph $G(V, E)$ is such that V_1 is of degree 1, V_2 is of degree 2, V_3 vertices of degree 3 and V_5 vertices of degree 4. Can such a graph exist? [3 marks]
- c. Given that the graph K_n has 21 edges.

- i. Find the number of vertices that K_n is composed of. [2 marks]
 - ii. Determine the degree of each vertex. [2 marks]
 - iii. Calculate the sum of the degrees of all its vertices. [2 marks]
- d. Graph G is represented by the following adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- i. Draw the graph G . [3 marks]
- ii. Determine whether G is a tree. Justify your answer. [2 marks]
- iii. Determine whether G is Eulerian graph. Justify your answer. [2 marks]

QUESTION FIVE [20 MARKS]

- a. Define the expected value (or expectation) of the random variable X (s) on the sample space S . Suppose X is the number that comes up when a die is rolled. What is the expected value of X ? [4 marks]
- b. State the generalized pigeon hole principle and show that, among 100 people, at least 9 of them were born in the same month. [4 marks]
- c. Consider the convention used in assigning a number plate to vehicles in Kenya, where a valid number plate has two parts. The first part consists of Three (3) Alphabetical Letters of which the first letter is **K** followed by two letters orderly selected from $[A-Z]$ {except I and O. the second part consist of three (3) Numbers orderly selected from $[0-9]$ and a Letter $[A-Z]$ {except I and O}.

Example **KDC 671Q**

How many number plates will be issued before abolishing this convention? [4 marks]

- d. Differentiate between N-P hard and NP-complete problem. [2 marks]
- e. Arrange the following expressions by growth rate from slowest to the fastest and state the value of n of which expression will be most efficient.

$4n^2$,	$\log_3 n$,	$n!$,	3^n ,	$2 \log_2 n$
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[6 marks]