(KNOWLEDGE FOR DEVELOPMENT)

KIBABII UNIVERSITY

(KIBU)

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SPECIAL/SUPPLEMENTARY EXAMINATIONS

FIRST YEAR FIRST SEMESTER

FOR THE DEGREE IN

(INFORMATION TECHNOLOGY/ COMPUTER SCIENCE)

COURSE CODE: BIT 111/CSC 112

COURSE TITLE: DISCRETE STRUCTURES

DATE: 27/09/2021 **TIME: 2.00 PM-4.00 P.M**

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE (COMPULSORY) [30 MARKS]

a. List the elements of each set where $N = \{1, 2, 3, \ldots\}$.

i. $A = \{x \in N \mid 3 < x < 9\}$ **[1 mark]**

ii. $B = \{x \in N \mid x \text{ is even}, x < 11\}$ **[1 mark]**

iii. $C = \{x \in N \mid 4 + x = 3\}$ **[1 mark]**

b. Given that set $A = \{x_1, x_2\}$ and set $B = \{y_1, y_2\}$. Define the cross product of A and B and show that Cartesian product is not commutative. **[3 marks]**

c. Let $U = \{1, 2, \ldots, 9\}$ be the universal set, and let $A = \{1, 2, 3, 4, 5\}$, $C = \{5, 6, 7, 8, 9\}$, $E = \{2, 4, 6, 8\}$,

d. $B = \{4, 5, 6, 7\}$, $D = \{1, 3, 5, 7, 9\}$ and $F = \{1, 5, 9\}$.

Find:

i. $A \cup B$ and $A \cap B$ **[2 marks]**

ii. $A \cup C$ and $A \cap C$ **[2 marks]**

iii. $D \cup F$ and $D \cap F$. **[2 marks]**

e. Prove by the method of induction that for all $n \in N$ then,

$$\frac{1}{3*5}+\frac{1}{5*7}+\frac{1}{7*9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

[4 marks]

f. Find the inverse (f^{-1}) of $f(x) = 4x^3 - 7$ **[2 marks]**

g. Find a counterexample for each statement were $U = \{3, 5, 7, 9\}$ is the universal set:

i. $\forall x, x + 3 \geq 7$ **[1 mark]**

ii. $\forall x, |x| = x$ **[1 mark]**

h. Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements: $p \wedge q$; $q \vee \neg p$ **[2marks]**

i. Evaluate the following

i. $C_{(11, 5)}$ **[2 marks]**

ii. Value of n if $P(n, 2) = 72$. **[2 marks]**

j. A survey of 80 car owners shows that 24 own a foreign-made car and 60 own a domestic-made car. Find the number of them who own:

i. only a foreign made car; **[2 marks]**

ii. only a domestic made car. **[2 marks]**

QUESTION TWO [20 MARKS]

a. i. Using Euclidean algorithm find the GCD and LCM of 1415 and 612. **[4 marks]**

ii. Find the value of x and y in **x**(1415)+ **y**(612)=gcd(1415, 612). **[4 marks]**

b. Suppose the only clothes you have are 2 t-shirts, 4 pairs of jeans and 6 pairs of shoes. In how many combinations you can choose a t-shirt, a pair of jeans and a pair of shoes? **[4 marks]**

c. Prove using mathematical induction that:

$1.2.3+2.3.4+3.4.5+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$

[4 marks]

d. Let p denote "Henry eats halibut," q denote "Catherine eats kippers," and r denote "I'll eat my hat."

i. Write a proposition that reads "If Henry eats halibut but Catherine does not eat kippers, then I'll eat my hat." **[2 marks]**

ii. Write the converse, inverse, and contrapositive of the statement "If Sally finishes her work, she will go to the basketball game." **[2 marks]**

QUESTION THREE **[20 MARKS]**

a. Give the universal set U representing the set of English alphabets, A a set of distinct elements of the word "**crocodile** ", B a set of distinct elements of the word "**continuous** " and C a set of distinct elements of the word "**myogenic**". Find:

i. A-C **[1 mark]**

ii. $(A\cup B\cup C)^c$ **[2 marks]**

iii. $|A\cup B|$ **[1 mark]**

iv. $A\cap B$ **[1 mark]**

b. Of 100 students in a university department, 45 are enrolled in English, 30 in History, 20 in Geography, 10 in at least two of three courses and just 1 student is enrolled in all three courses.

i. Represent these information on a Venn diagram **[4 marks]**

ii. How many students take none of these courses? **[2 marks]**

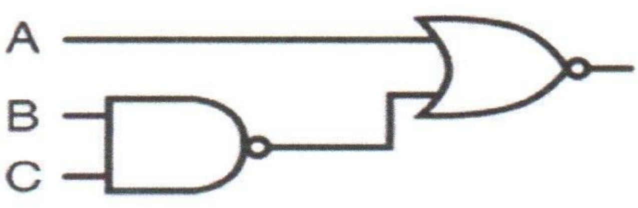
c. In a class of 30 students, 10 got A on the first test, 9 got A on a second test, and 15 did not get an A on either test. Find: the number of students who got:

i. an A on both tests; **[2 marks]**

ii. an A on the first test but not the second; **[2 marks]**

iii. an A on the second test but not the first. **[2 marks]**

d. State the output of the following circuit. **[3 marks]**



QUESTION FOUR **[20 MARKS]**

a. Given sets A, B and C such that all are non-empty sets. State the inclusive- exclusive principle. **[2 marks]**

b. Let A = {1, 2, 3, 4, 5}. Determine the truth value of each of the following statements:

i. $(\exists x\in A)(x+3=10)$ (c) $(\exists x\in A)(x+3<5)$ **[2 marks]**

ii. $(\forall x \in A)(x + 3 < 10)$ (d) $(\forall x \in A)(x + 3 \leq 7)$ **[2 marks]**

c. Determine the truth value of each of the following statements where U = {1, 2, 3} is the universal set: **[3 marks]**

   i. ∀x∃y, x2 + y2 < 12
   ii. ∀x∀y, x2 + y2 < 12

d. Give the $f(x) = \frac{x+1}{x^2}$, $g(x) = 4x^2+7$ and $h(x) = \frac{x^2-1}{x+1}$ find:

   i. Domain and range of f(x) and h(x) **[2 marks]**
   ii. The inverse $g^{-1}(x)$ of g(x) **[3 marks]**
   iii. Is g(x) bijective? Explain. **[2 marks]**
   iv. f(g(h(x))) **[2 marks]**
   v. g(h(2)) **[2 marks]**

QUESTION FIVE [20 MARKS]

a. Using relevant examples differentiate between a function and a relation. **[2 marks]**

b. Let A = {2, 3, 4, 5 }and let R = {(2, 3), (3, 3), (4, 5), (5, 1)}.Is R symmetric, asymmetric or antisymmetric? **[2 marks]**

c. Let A = {1, 2, 3}, B = {a, b, c}, and C = {x, y,z}. Consider the following relations R and S from A to B and from B to C, respectively. R = {(1, b), (2, a), (2, c)} and S = {(a, y), (b, x), (c, y), (c, z)}

   (i) Find the composition relation R∘S. **[2 marks]**

   (ii) Find the matrices MR, MS, and MR∘S of the respective relations R, S, and R∘S, and compare MR∘S to the product MRMS. **[2 marks]**

d. Let A, B, C and D be sets. Suppose R is a relation from A to B, S is a relation from B to C and T is a relation from C to D. Then show that (R∘S)∘T = R∘(S∘T). Let R be the relation on N defined by x + 3y = 12, i.e. R = {(x, y)| x + 3y = 12}.

   i. Write R as a set of ordered pairs. (c) Find R−1. **[2 marks]**
   ii. Find the domain and range of R. (d) Find the composition relation R∘R. **[2 marks]**

e. A women student is to answer 10 out of 13 questions. Find the number of her choices where she must answer:

   i. the first two questions **[2 marks]**
   ii. exactly 3 out of the first 5 questions **[2 marks]**
   iii. the first or second question but not both **[2 marks]**
   iv. at least 3 of the first 5 questions. **[2 marks]**