(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MAT 110

COURSE TITLE: BASIC CALCULUS

DATE: 16/02/2021 **TIME:** 8 AM - 10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

(a) Define the following terms
   i. Tangent line
   ii. Function
   iii. Kinematics
   iv. Acceleration (4 marks)

(b) Evaluate $\lim_{t\to\infty} \frac{2t^2}{3t^3+t-1}$ (3 marks)

(c) Given $G(x) = x^2 - 3x + 7$ evaluate $G(b + 1)$ (2 marks)

(d) Find the domain and range of $f(t) = \sqrt{1 + 5t}$ (2 marks)

(e) Given $f(t) = t^2 + 1$, $g(t) = \frac{3}{t}$ and $h(t) = 2t$, determine the following composite functions;
   i. $g(h(t))$
   ii. $f(g(t))$ (4 marks)

(f) Find y'' if $x^4 + y^4 = 1$. (4 marks)

(g) Find the derivative of $y = x^3 + x + 1$ from the 1st principle or using the delta method. (4 marks)

(h) Differentiate;
   i. $y = \frac{3cos\,(2x)}{x^3}$ (3 marks)
   ii. $y = (1 + x^2)^5\, ln\, x^2$ (2 marks)
   iii. $x = \frac{4t^{\frac{3}{2}}}{3}$ (2 marks)

QUESTION TWO (20 MARKS)

(a) Find the derivative of $y = x^3\, cos2x\, ln\, x$ (4 marks)

(b) If the acceleration of a moving particles is $a(t) = 18t - 2$, find its velocity and displacement at any time t. (4 marks)

(c) The position of a particle is given by $S = t^3 - 6t^2 + 9t$ where t is time in seconds and S is displacement in metres,
   i. Find the velocity and acceleration at any time t. (2 marks)
   ii. What is the velocity and acceleration after 5 seconds? (2 marks)
   iii. When is the particle at rest? (2 marks)

(d) If a ball is thrown vertically up with velocity of $80m/s$ such that its height after time t seconds is $S = 80t - 16t^2 + 10$.
   (i) Calculate time taken by the ball to reach the maximum height. (4 marks)
   (ii) What is the maximum height reached by the ball? (2 marks)

QUESTION THREE (20 MARKS)

(a) Differentiate between a maximum point and a minimum point. (2 marks)

(b) Find the critical point of $y = x^3 - 3x^2 - 9x + 2$ then determine whether each critical point is local minimum or local maximum point. (7 marks)

(c) Find the critical point of $g(x) = x^3 - 12x, -\infty \leq x \leq \infty$ and identify the interval on which the function g in increasing or decreasing using the first derivative test. Sketch it. (6 marks)

(d) Find the coordinate points on the following curve at which the gradient is 2.

$$y = 2x - x^2$$

(3 marks)

(e) Determine whether the following functions are even, odd or neither even nor odd.

i. $f(x) = x^3 + x$

ii. $g(x) = -x^4 + 1$ (2 marks)

QUESTION FOUR (20 MARKS)

(a) Find the equation of the tangent and normal to the curve $x = t^2 - 4t + 1$ at the point $(-2,13)$. (5 marks)

(b) State the Rolle's theorem, hence find the value of c satisfying the conclusion of Rolle's theorem for $f(x) = \frac{x^3}{3} - 3x$ on the interval $(-3,3)$ (5 marks)

(c) If $y = \frac{\sin x}{x}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (5 marks)

(d) Given $x(t) = 5t^2,\ y(t) = 64t - 16t^2$

i. find $\frac{dy}{dx}$

ii. the point at which the curve has a horizontal tangent

iii. the point at which the curve has a vertical tangent (5 marks)

QUESTION FIVE (20 MARKS)

(a) Find the slope of the line tangent to the graph of the equation $x^3 + y^3 - 2xy^2 + yx + 2y = 1 + y^2$ at the point $(0,1)$ (4 marks)

(b) Differentiate

i. $y = (2x^2 + 1)^2(x^3 - x + 1)^4$

ii. $y = e^{(2x^3+1)^2}$ (6 marks)

(c) Find

i. $\lim_{x \to 1} \frac{x^2-3x+2}{x^2-5x+4}$

ii. $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$ (5 marks)

(d) Find the general antiderivative $f(x)$ of the following

i. $f'(x) = \frac{x^4-x^2+1}{x^2}$

ii. $f'(x) = \cos x + \sin x$ (5 marks)