(KNOWLEDGE FOR DEVELOPMENT)

KIBABII UNIVERSITY

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UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SPECIAL/SUPPLEMENTARY EXAMINATIONS

FIRST YEAR SECOND SEMESTER

FOR THE DEGREE IN

(COMPUTER SCIENCE)

COURSE CODE: CSC 121

COURSE TITLE: DISCRETE STRUCTURES II

DATE: 29/01/ 2021 **TIME: 8.00 A.M. – 10.00 A.M.**

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE [30 MARKS]

a. Represent the proposition $(\sim A \vee B) \rightarrow (C \wedge \sim B)$ by its truth table. [4 marks]

b. Write the negation of the statement: $(\exists x)\ (\forall y)\ p(x, y)$. [2 marks]

c. What is the probability that when two dice are rolled, the product of the numbers on the two dice is less that 10? [3 marks]

d. Write the following statement in symbolic form using quantifiers:

(i) All students have taken a course in logic [2 marks]

(ii) Some students are intelligent, but not hardworking [2 marks]

e. Let A = {1, 2, 3, 4, 5}, determine the truth value of the following:

(i) $(\forall x \in A)(x + 7 = 10)$ [2 marks]

(ii) $(\exists x \in A)(x + 7 < 5)$ [2 marks]

f. Translate the following sentences into a formula in propositional logic: "If Mr. Holmes told the truth and Mr. Watson did not hear anything, then it cannot be both that the butler did it and that the butler returned to his hotel room that night." [3 marks]

g. Given two integers **a** and **b** such that a=1529 and b=14038, find the Gcd(a, b). hence or otherwise find the values of integers **s** and **t** such that **x(a)+y(b)=Gcd(a, b)** [6 marks]

h. Explain the application of **The Chinese remainder theorem** and **Little Fermat' s theorem** as used in the study of discrete structures. [4 marks]

QUESTION TWO [20 MARKS]

a. Define the expected value (or expectation) of the random variable X (s) on the sample space S. [3 marks]

b. Let X be the number that comes up when a die is rolled. What is the expected value of X? [3 marks]

c. What is the variance of the random variable X, where X is the number that comes up when a die is rolled? [4 marks]

d. A population of a country in 2019 was 49.7 million. Given that it is growing at a rate of 1.1 per year.

i. Set up a recurrence relation for the population in n years after 2020. [3 marks]

ii. Find the explicit formula for the population n year after 2020. [4 marks]

iii. What will be the population in 2030? [3 marks]

QUESTION THREE [20 MARKS]

a. How many edges are there in a graph with 10 vertices each of degree 5? [3 marks]

b. Does there exists a simple graph with the degree sequence 3 , 3 , 3 , 2 ? [3 marks]

c. Find the number of edges and vertices in $K_{50,100}$ [3 marks]

d. Graph G is represented by the following adjacency matrix A

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

i. Draw the graph. [3 marks]

ii. Determine whether G is Hamiltonian graph. Justify your answer. [3 marks]

iii. Determine whether G is Eulerian graph. Justify your answer. [3 marks]

iv. Determine whether G is a tree. Justify your answer [2 marks]

QUESTION FOUR [20 MARKS]

a. Define the following terms as used in the study of discrete structures

i. Linear Congruence [1 marks]

ii. Equiprobable Spaces [1 marks]

iii. Random Variables [1 marks]

iv. Independent Event [1 marks]

b. A pair of dice is loaded. The probability that a 4 appears on the first die is 2/7, and the probability that a 3 appears on the second die is 2/7. Other outcomes for each die appear with probability 1 /7. What is the

probability of 7 appearing as the sum of the numbers when the two dice are rolled? [4 marks]

c. What is the probability that the numbers 11, 4, 17, 39, and 23 are drawn in that order from a bin containing 50 balls labeled with numbers 1, 2, . . ., 50 if:

i. The ball selected is not returned to the bin before the next ball is selected and; [4 marks]

ii. The ball selected is returned to the bin before the next ball is selected? [4 marks]

d. There are many lotteries now that award enormous prizes to people who correctly choose a set of six numbers out of the first n positive integers where n is usually between 30 and 60. What is the probability that a person picks the correct six numbers out of 40? [4 marks]

QUESTION FIVE [20 MARKS]

a. Given the arithmetic sequence where $a1 = 6$ and $a5 = -6$. What is $a3$? [2 marks]

b. Find the solution of the linear homogeneous recurrence relation of second-degree with constant coefficients $an = 7a_{n-1} - 10a_{n-2}$, $a_0 = 2$, $a_1 = 3$. [5 marks]

c. Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n = a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \ldots$, and suppose that $a_0 = 3$ and $a_1 = 5$. What are a_2 and a_3? [3 marks]

d. Use mathematical induction to show that $2 + 4 + 6 + \cdots + 2n = n^2 + n$, for $n \geq 1$ [4 marks]

e. Find a positive integer *(a)* such that when *(a)* is divided by **3** it gives a remainder of **2**, when divided by **5** remainder is **4** and when divided by **7** remainder is **6**. [6 marks]