(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 303

COURSE TITLE: LINEAR ALGEBRA III

DATE: 17/02/2021 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30MARKS)

(a). Define the following terms

(i). Real inner product function **(4 Marks)**

(ii). Hermitian matrix **(1 Mark)**

(iii). Nilpotent matrix **(1 Mark)**

(iv). A bilinear function **(2 Marks)**

(b). Prove that an orthogonal matrix is Isometric. **(4 Marks)**

(c). Find the eigenvalues of matrix A.

$$A = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}$$ **(4 Marks)**

(d). Make a change of variable to transform the quadratic form $Q(x) = 2x_1^2 - 6x_1x_2 + 2x_2^2$ into a quadratic form with no cross-product terms. **(7 Marks)**

(e). Prove the eigenvectors in different eigenspaces of a symmetric matrix are orthogonal.

(4 Marks)

(f). Let λ be an eigenvalue of a real $n \times n$ matrix, B, and x the corresponding eigenvector. Show that if $\bar{\lambda}$ is also an eigenvalue of B and $\bar{x}$ is a corresponding eigenvector. **(3 Marks)**

QUESTION TWO (20 MARKS)

a). Define a unitary matrix hence show that it is Isometric. **(5 Marks)**

b). Let P and Q be linear transformations on complex vector space V such that $P: V \rightarrow V$ and $: V \rightarrow V$. Prove that

(i). $(P + Q)^* = P^* + Q^*$ **(3 Marks)**

(ii). $(PQ)^* = P^*Q^*$ **(3 Marks)**

(iii). $(kP)^* = \bar{k}P^*$ **(3 Marks)**

c). Prove that the orthogonally diagonalizable matrix has an orthonormal set of n eigenvectors.

(6 Marks)

QUESTION THREE (20 MARKS)

(a). Find Euclidean inner product $\langle u, v\rangle$, and the norm $\|u\|$ where $u = (1 + 4i, -2i, 3i - 1)$ and $v = (1 - 4i, 2i, 3i - 1)$. **(4 Marks)**

(b). Let $\mathbb{C}^n$ be a complex vector space $u, v \in \mathbb{C}^n$. If $\bar{u}, \bar{v}$ denotes the conjugates of u and v respectively. Prove that. $\overline{u + v} = \bar{u} + \bar{u}$. **(3 Marks)**

(d). Prove that a matrix N is nilpotent if and only if its eigenvalues are zero. **(7 Marks)**

(d). Determine all possible Jordan Canonical forms J for a linear operator $T: V \to V$ whose characteristic polynomial $\Delta(t) = (t - 4)^6$ and whose minimum polynomial $m(t) = (t - 4)^3$.

(6 Marks)

QUESTION FOUR (20 MARKS)

a).(i).What is an orthonormal set of vectors **(2 Marks)**

(ii). Prove that if A is $n \times n$ orthogonal matrix, then the column vectors of A forms an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product **(6 Marks)**

b).(i). Let A be $n \times n$ matrix over K. Show that the mapping f defined by $f(X, Y) = X^T AY$ is a bilinear form on K^n. **(5 Marks)**

(ii). Find a quadratic form corresponding to the following symmetric matrix

$$A = \begin{bmatrix} 1 & -2 & 5 \\ -2 & 16 & -4 \\ 5 & -4 & 4 \end{bmatrix}$$ **(2 Marks)**

(iii). Hence, classify the matrix as either positive definite, negative definite or indefinite. Show your working. **(5 Marks)**

QUESTION FIVE (20 MARKS)

Orthogonally diagonalize matrix A, $A = \begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$ **(20 Marks)**