(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 110**

COURSE TITLE: **BASIC CALCULUS**

DATE: 05/02/2021 **TIME:** **8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30marks)

(a). i Define a function (1mk)

ii. Find the domain of the function $f(x) = \sqrt{x+2}$. (2mks)

iii. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$, find the function $g \circ f$ and its domain (2mks)

(b) i. State precisely the definition of a limit. (2mks)

ii. Prove that $\lim_{x \to 0^+} \sqrt{x} = 0$. (3mks)

(c) Prove that $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$. (4mks)

(d) Define a continous function (1mks)

(e) Where are the following functions continuous?

i. $h(x) = \sin(x^2)$ ii. $F(x) = \frac{1}{\sqrt{x^2+7-4}}$ (4mks)

(f) Find the derivative of the function $f(x) = \frac{2x+3}{x-2}$ from first principles and hence find the equation of the tangent to the curve $f(x) = \frac{2x+3}{x-2}$ at the point$x = 3$. (5mks)

(f) Differentiate the functions:

i. $f(t) = \sqrt{t}(a + bt)$ (2mks)

ii. $y = \sqrt{secx^3}$ (2mks)

(g) .If $x^2 + y^2 = 25$. Find $\frac{dy}{dx}$. (3mks)

(h).Find an equation of the tangent line to the curve $y = \frac{\sqrt{x}}{1+x^2}$ at the point$(1, {}^1/_2)$. (3mks)

QUESTION TWO (20MKS)

(a) (i) State Rolle's theorem (2mks)

(ii) Find the value of c prescribed in the Rolle's Theorem for the function $f(x) = \frac{x^3}{3} - 3x$ on the interval $-3 \leq x \leq 3$ and show that $a = -3$ and $b = 3$. (3mks)

b) Prove that the equation $x^3 + x - 1 = 0$ has exactly one root (3mks)

(c) A closed cylindrical jam tin is of height h cm and radius $r$cm. Its total surface area is $A\ cm^2$ and its volume$V cm^3$. Find an expression for A in terms of r and h. Taking $= 48\pi$, find

i. An expression for h in terms of r and hence find an expression for V in terms of r. (2mks)

ii. The value of r which will make V a maximum. (3mks)

(d) Show that if f is differentiable at a then f is continous. (4mks)

(e) Differentiate $f(x) = \frac{secx}{1+tanx}$ (3mks)

QUESTION THREE (20 MKS)

a) A function f is defined by $f(x) = \begin{cases} 1-x & if \quad x \leq 1 \\ x^2 & if \quad x > 1 \end{cases}$ .Evaluate $f(0), f(1)$ and $f(2)$ and sketch the curve. (3mks)

b) Define

i. an even function

ii. odd function (2mks)

c) Determine whether each of the following functions is even, odd or neither even nor odd.

i. $f(x) = x^5 + x$

ii. $g(x) = 1 - x^4$

iii. $h(x) = 2x - x^2$ (3mks)

(d) Given $F(x) = cos^2(x + 9)$, find functions f, g and h such that $F = f \circ g \circ h$ (3mks)

(e) Prove that $\lim_{x \to 0^+} \sqrt{x} = 0$ (3mks)

(f) Evaluate

i. $\lim_{\theta \to 0} \frac{cos\theta - 1}{\theta}$ (3mks)

ii. $\lim_{x \to \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$ (3mks)

QUESTION FOUR (20 MKS)

(a) Establish the point at which the function

$$f(x) = \begin{cases} \frac{x^2-x-2}{x-2} & if \quad x \neq 2 \\ 1 \quad if & x = 2 \end{cases}$$ is discontinuous. (2mks)

(b) If f and g are continous at a and c is a constant, show that the function $f + g$ is continous at a. (3mks)

(c) Given $f(x) = \frac{2x}{x-3}$, compute both the left and right limits for f at $x = 3$. Make comments about the function and sketch it. (4mks)

(d) Find the minimum and maximum points on the curve $f(x) = x^3 - 4x^2 - 3x + 2$ and classify them. (4mks)

(e) Find the derivative of y with respect to x given

i. $x^2y - xy^2 + x^2 + y^2 = 0$ (2mks)

ii. $y = e^{lnx^2}$ (2mks)

iii. $y = cos^{-1}\left(\frac{1-x}{1+x}\right)$ (3mks)

QUESTION FIVE (20 MKS)

(a) Find the general antiderivative of

i. $\frac{1}{x^3}$ ii. $sin4x - 5sin5x$ (3mks)

(b) A farmer has $2400ft$ of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fencing along the river. What are the dimensions of the field that has the largest area?. (6mks)

(c) Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection and local maxima and minima. Use the information to sketch the curve. (7mks)

(d) Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing. (4mks)