(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: STA 446

COURSE TITLE: BAYESIAN STATISTICS

DATE: 08/02/2021 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)(COMPULSORY)

1. (a) Differentiate between informative and non informative priors. (2 mks)

   (b) Explain the situation when the posterior distribution of θ can be equivalent to the likelihood function. (2 mks)

   (c) Give an expression for the Bayes rule for the conditional $P(y|x)$ and state the importance of the denominator (Assume X and Y are continuous random variables) (3 mks)

   (d) In 40 tosses of a coin, 24 heads were obtained. Find the Bayes estimate with squared error loss function for

      i. p (4 mks)

      ii. $\frac{1}{p}$ (4 mks)

   (e) In order to determine how effective a magazine is reaching its target audience, a market research company selects a random sample of people from the target audience and interviews them. Out of 150 people in the sample, 29 had seen the latest issue.

      i. What is the distribution of y, the number who have seen the latest issue? (2 mks)

      ii. Use the uniform prior for π, the proportion of the target audience that has seen the latest issue. What is the posterior distribution of π? (3 mks)

   (f) Let $X \sim B(n,p)$. Assume the prior distribution of p is uniform on [0,1]. Show that the posterior is essentially the likelihood function. (5 mks)

   (g) A bag containing a total of 7 balls, some of which are red and the rest of which are green. Let the random variable X be the number of red balls in the urn. Find the Bayesian estimate of X (5 mks)

QUESTION TWO (20 MARKS)

2. (a) Suppose the binomial pdf describes the number of votes a candidate might receive in a poll conducted before the general election. Moreover, suppose a beta prior distribution has been assigned to θ, and every indicator suggests the election will be close. The pollster, then, has good reason for concentrating the bulk of the prior distribution around the value $\theta = \frac{1}{2}$. Setting the two beta parameters r and s both equal to 135 will accomplish that objective (in the event $r = s = 135$, the probability of θ being between 0.45 and 0.55 is approximately 0.90).

    i. Find the corresponding posterior distribution. (5 mks)

    ii. Find the squared-error loss Bayes estimate for θ and express it as a weighted average of the maximum likelihood estimate for θ and the mean of the prior pdf. (5 mks)

   (b) Let $Y|\pi$ be binomial $(n = 4, \pi)$. Suppose we consider that there are only three possible values for π, 0.4, 0.5 and 0.6 and that $Y = 3$. Find the posterior probability distribution of π (10 mks)

QUESTION THREE (20 MARKS)

3. (a) The random variable X has a Poisson distribution with unknown parameter λ. It has been determined thay λ has the subjective prior probability function given in the table below. A random sample of size 3 yields the X-values 2,0, and 3.

λ	1.0	1.5	2.0
$\pi(\lambda)$	1/3	1/2	1/6

    i. Identify the prior used in this problem (2 mks)

    ii. Find the likelihood of the data in $f(x|\lambda)$ (2 mks)

    iii. Find the posterior distribution of λ (10 mks)

   (b) Suppose $X_1, ..., X_n$ is a sample from geometric distribution with parameter p, $0 \leq p \leq 1$. Assume that the prior distribution of p is beta with $a = 4$ and $b = 4$. Find

i. the posterior distribution of p (3 mks)

ii. the Bayes estimate under quadratic loss function (3 mks)

QUESTION FOUR (20 MARKS)

4. (a) Suppose that X is a geometric random variable, where $P_X(k|\theta) = (1-\theta)^{k-1}\theta, k = 1, 2, \ldots$. Assume that the prior distribution for θ is the beta p.d.f. with parameters α and β. Find the posterior distribution for θ. (6 mks)

   (b) Suppose the binomial pdf describes the number of votes a candidate might receive in a poll conducted before the general election. Moreover, suppose a beta prior distribution has been assigned to θ, and every indicator suggests the election will be close. The pollster, then, has good reason for concentrating the bulk of the prior distribution around the value $\theta = \frac{1}{2}$. Setting the two beta parameters r and s both equal to 135 will accomplish that objective (in the event $r = s = 135$, the probability of θ being between 0.45 and 0.55 is approximately 0.90).

   i. Find the corresponding posterior distribution. (3 mks)

   ii. Find the squared-error loss Bayes estimate for θ and express it as a weighted average of the maximum likelihood estimate for θ and the mean of the prior pdf. (3 mks)

   (c) X is a Bernoulli random variable with success probability θ, which is known to be either 0.3 or 0.6. It is desired to test the null hypothesis $H_0 : \theta = 0.3$ against the alternative $H_1 : \theta = 0.6$ using a Bayes 0.05 test assuming the vague prior probability distribution for θ: $P(\theta = 0.3) = P(\theta = 0.6) = 0.5$. A sample of 30 trials on X yields 16 successes.

   i. Find the posterior probability of the null hypothesis (5 mks)

   ii. Check the rejection criterion of the Bayes 0.05 test (3 mks)