(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

COURSE CODE: **MAT 811**

COURSE TITLE: ABSTRACT INTERGRATION I

DATE: 15/02/2021 **TIME: 2 PM -5 PM**

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE

a) Define the following terms: (5marks)

i. Ring
ii. δ- Ring
iii. Monotone class
iv. Measure
v. Borel function

b) Given R is a ring of subset of a set x. Show that $M(R) = G(R)$ i.e the monotone class coincides with the ring generated by the ring R. (5marks)

c) Show that if a measure on a ring R and M is the class of all M^*- measurable sets, then $G(R) \leq M$ and the restriction of M^* to $G(R)$ is a measure $\overline{M}$ extending$M$ (7marks)

d) State the 'Unique extension theorem' U.E.T (3marks)

QUESTION TWO

a) Define the following terms:

i. Measurable space (1mark)
ii. Locally measurable (1mark)
iii. Measurable functions (2marks)
iv. Characteristics functions (2marks)

b) Show that in order that a function $f: x \to \mathbb{R}$ be measurable , it is necessary and sufficient that

i. $N(f)$ be measurable and
ii. $f^{-1}(M)$ be locally measurable for every borel set M. (7marks)

c) Let f_n be a sequence of measurable functions. Suppose that for each point in x in X, the sequence $f_n(x)$ is bounded below so that $g = GLB\ f_n$ is real – valued. Then the function g is also measurable. Similarly, if the sequence $f_n(x)$ is bounded above for each point x, then show that the function $h = LUB\ f_n$ is measurable. (7marks)

QUESTION THREE

a) Suppose f and g are simple functions, c is a real number and A is locally measurable set, then show that all of the following functions are simple (8marks)

i. Cf
ii. $f + g$
iii. $l + l$
iv. $f \cup g$
v. $f \cap g$
vi. f^+, f^-
vii. X_A, f
viii. fg

b) Suppose f is a measuarable function, α is a real number and $c > 0$, then show that $f \cap c$ is a measurable function. (6marks)

c) Show that if f is a measurable function, then there exists a sequence of simple functions f_n such that f_n converges to f pointwise on X, that is $f_n(x) \to f(x)$ for each x in X. If more over $f \geq 0$, one can make $0 \leq f_n \uparrow f$. (6marks)

QUESTION FOUR

a) Show that in a finite measure space (x, J, m) . The measure M is necessarily bounded that is the number $L \cup B\{M(E): E \in J\}$ (8marks)

b) Show that

i. If $f_n \to f\ a.e$ then f_n is fundamental a.e (3marks)

ii. If $f_n \to f\ a.e$ and $f_n \to g\ a.e$ than $f = g\ a.e$ (3marks)

iii. If $f_n \to f\ a.e$ and g is a real valued function such that $f_n \to g\ a.e$ and $f_n \to f\ a.e$ (3marks)

iv. If f_n, f, g are real valued functions $f_n \to f\ a.e$ and $f_n \leq g\ a.e$ for each n then $f \leq g\ a.e$ (3marks)

QUESTION FIVE

a) Show that if f_n is a sequence of ISF (untegrable simple funvctions) such that $f_n \downarrow 0$ then $l(f_n) \downarrow o$ (5marks)

b) Show that if $f \geq 0$ and $g \geq 0$ are integrable, $c \geq 0$ is a real number and A is a locally measurable set then

i. cf is integrable and $\int cf\ du = c \int f du$ (5marks)

ii. $f + g$ is integrable and $\int (f + g) du = \int fgu + \int gdu$ (5marks)

c) Suppose $0 \leq f_n \uparrow f$ and $0 \leq g_n \uparrow f$, where f is measurable, f_n and g_n are sequences of ISF and $l(f_n)$ is bounded. Then show that $l(g_n)$ is bounded and $L \cup B_n l(f_n) = L \cup B_n\, I(g_n)$ (5marks)