(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAA 111**

COURSE TITLE: **DIFFERENTIAL CALCULUS/CALCULUS I**

DATE: 15/02/2021 **TIME:** 2 PM - 4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE COMPULSORY (30 MARKS)

a) Given $g(x) = x^2 + 2x$, $f(x) = -x^2 + 4x$ and $h(x) = x - 2$

find $hogoh(x)$ (2mks)

b) Evaluate the following limits

i. $\lim_{x\to 9} \frac{x^2-81}{\sqrt{x}-3}$ (3mks)

ii. $\lim_{x\to 2} \frac{3x^2-x-10}{x^2-4}$ (3mks)

c) Evaluate

$$\lim_{x\to 0} \frac{\tan \alpha x}{\sin \beta x}$$ (3mks)

d) Prove that: $\lim_{x\to 3} 4x - 5 = 7$ (3mks)

e) Determine if the following function is continuous at $x = 3$

$$f(x) = \begin{cases} x^2 - 4 & x < 3 \\ 5 & x = 3 \\ x + 2 & x > 3 \end{cases}$$ (4 mks)

f) Find the derivative of

$y = \tan(2x)e^{ln(2x^3+5x)}$ (4mks)

g) Find the equation of the tangent and normal to the curve

$y = 3x^2 - 2x - 5$ at the point (3,11) (5mks)

(i) Differentiate $y = \frac{1-sinx}{cosx}$ (3mks)

QUESTION TWO (20 MARKS)

a) Prove the following theorems

i. $\lim_{\alpha\to 0} \frac{sin\alpha}{\alpha} = 1$ (5mks)

ii. $\lim_{\theta\to 0} \frac{cos\theta-1}{\theta} = 0$ (5mks)

b) Evaluate the following limits

$\lim_{x\to 0} \frac{1-cos2x}{xsinx}$ (5mks)

c) A square iron sheet measures 20M on a side is to be used to make an open top box by cutting a small square iron sheet from each corner and bending up the sides. How large a square should be cut from each corner in order that the box shall have as large volume as possible? (5mks)

QUESTION THREE (20 MARKS)

a) Find from the 1st principles or using the delta method the derivative of

$$y = 3x^3 - 2x^2 + 2x + 4$$ (6mks)

b) Find the slope of the line tangent to the graph of the equation

$6x^3 + y^3 - 2xy^2 + yx^2 + 2y = 1 + y^2$ at the point $(0.5, -0.6)$ (5mks)

c) Find $\frac{dy}{dx}$ if $y = x(x+3)^4$ (4mks)

d) Find the Cartesian equation for each of the following parametric form.

$x = 2t - 1; y = 1 - t^2$ (5mks)

QUESTION FOUR (20 MARKS)

a) Find the equation of the line tangent to the graph of $x(t) = t^2 + 1$ and $y(t) = \sqrt{1+t}$ at the point $t = 3$ (10mks)

b) If $y = \frac{sinx}{x^2}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ hence prove that

$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 + 2)y = 0$ (10mks)

QUESTION FIVE (20 MARKS

a) The distance S moved by a body in t seconds is given by $S = 2t^3 - 13t^2 + 24t + 10$ find

(i) The velocity when $t = 4$ seconds (3 mks)

(ii) The value of t when the body comes to rest (3 mks)

(iii) Find acceleration at 4 seconds (3 mks)

b) Find the coordinates of any stationary points on the curve $y = x^3 - 2x^2 - 4x$ and distinguish between these points. (6mks)

c) An open rectangular box with volume 2 M^3 has a square base. Express the surface area of the box as a function of the length of a side of the base. Hence find the dimensions. (5mks)