(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 252**

COURSE TITLE: **ENGINEERING MATHEMATICS II**

DATE: 08/02/2021 **TIME: 8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

a) Show that the equation $x^3 + 3x^2 - 4 = 0$ has a root between 3 and 4 (4 mks)

b) Show using Newton Raphson method that if the solution of the equation in (a) above is x_n, then a better approximation x_{n+1}, is given by $x_{n+1} = \frac{2x_n^3 - 3x_n^2 + 4}{3x_n^2 - 6x_n}$ (8 mks)

c) Taking $x_1 = 3.4$, use the formula in (b) above to find the root of the equation in (a) above to four decimal places (7 mks)

d) For the forward difference operator Δ, show that for $f(x) = x^2 + 8x - 5$,

$$\Delta^2 f(x) = 2h^2$$ (6 mks)

e) The following values were found empirically

x	2.1	2.4	2.7	3.0	3.3	3.6
y	3.2	2.7	2.9	3.5	4.1	5.2

Use trapezoidal rule to estimate $\int_{2.1}^{3.6} y dx$ (5 mks)

QUESTION TWO (20 MARKS)

a) Use the Requla- Falsi method to find an approximate value of the equation $x\ log_{10}x - 1.2 = 0$ correct to 3 decimal places starting with $x_1 = 2$ and $x_2 = 3$ with two steps (7 mks)

b) Construct the backward finite difference table for

x	1	2	3	4	5
f(x)	4	6	9	12	17

(5 mks)

c) Given the iterative formula $x_{n+1} = 5 - \frac{2}{x_n}$

(i) Show that the formula converges using $x_1 = 4$ (3 mks)

(ii) Find $x_{2,}\ x_{3,}\ x_4$ correct to three significant figures (3 mks)

(iii) Find the equation being solved by this iterative formula (2 mks)

QUESTION THREE (20 MARKS)

a) Given the empirical data below

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

Find, using Newton Gregory interpolating formula the values of

(i) f(2.2) (5 mks)

(ii) f(6.3) (4 mks)

b) Use the mid ordinate rule with six equally spaced mid ordinates to find the area bounded by the curve $y = -x^2 + 49, the\ x - axis\ and\ the\ ordinates\ x = 0\ and\ x = 6$ (6 mks)

c) Given the operators $E\ and\ \nabla$, show that $E\nabla = \nabla$ (5 mks)

QUESTION FOUR (20 MARKS)

a) Use Simpson's $\frac{3}{8}$ rule with $n = 6$ to estimate $\int_0^3 \frac{1}{1+x} dx$ (7 mks)

b) Estimate the missing value in the table below using finite difference

x	1	2	3	4	5
f(x)	2	5	7	_	32

(7 mks)

c) Interpolate the value of the function corresponding to x=4 using Lagrange's interpolation formula from the data below

x	2	3	5	8	12
f(x)	10	15	25	40	60

(6 mks)

QUESTION FIVE (20 MARKS)

a) Apply Euler's method to find an approximate value for y corresponding to $x = 1.5$ given that $\frac{dy}{dx} = x + 2y \text{ and } y(1) = 1$ (6 mks)

b) For the table given below

x	3	7	9	10
f(x)	168	120	72	63

Find (i) the interpolating polynomial (6 mks)

(ii) using the interpolating formula the value of f(6) (2 mks)

c) Given the table of values below

x	1	1.2	1.4	1.6	1.8	2.0
f(x)	0	0.1	0.5	1.25	2.40	3.90

Find (i) $f^{1}(x)$ (3 mks)

(ii) $f^{11}(x)$ at the point $x = 1.1$ (3 mks)