(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 323**

COURSE TITLE: NUMERICAL ANALYSIS I

DATE: 12/02/2021 **TIME:** **11 AM -1 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION I (30 marks)

a) Find to three decimal places the root of the equation $x^3 - 5x - 11 = 0$ by the method of iteration. (5 marks)

b) Find the first term of the sequence whose second and subsequent terms are 8, 3, 0, -1, 0, … (4 marks)

c) Using the simple form of Newton's method, solve the equations (up to (x^2, y^2)).

$$f(x,y) = y^2 + 4x^2 + 2xy - y - 2 = 0$$

$$g(x,y) = y^2 + 2x^2 + 3xy - 3 = 0 \text{ starting with } x_0 = 0.4, y_0 = 0.9$$ (10 marks)

d) Use Lagrange's formula to fit a polynomial to the data:

x	-1	0	2	3
y	-8	3	1	12

and hence find y at $x = 1$. (4 marks)

e) i) Given that $y_3 = 2, y_4 = -6, y_5 = 8, y_6 = 9$ and $y_7 = 17$, calculate $\Delta^4 y_3$

(4 marks)

ii) Show that $\delta = E^{-1/2}\Delta = E^{1/2}\nabla$ where δ is the central difference operator, E is the shifting operator, Δ and ∇ are the forward and backward difference operators respectively.

(3 marks)

QUESTION 2 (20 marks)

a) Find the seventh term of term of the sequence 2, 9, 28, 65, 126, 217 and also find the general term. (5 marks)

b) Using the Newton-Raphson method, solve the equations $x^2 + y^2 = 16$ and $x^2 - y^2 = 4$ given that the starting solution is $(2\sqrt{2}, 2\sqrt{2})$ (2 iterations) (12 marks)

c) Convert the hexadecimal number 39.B8 to an octal number. (3 marks)

QUESTION 3 (20 marks)

a) Use values at x_0 and x_1 in the table below to get an interpolated value for $f(x) = \sin x$ at $x = 0.632$ radians using linear interpolation, and compute an error estimate for the interpolated value.

x	$x_0 = 0.5^c$	$x = 0.632^c$	$x_1 = 1.00^c$
$f(x) = \sin x$	0.47942554		0.84147099

(6 marks)

b) From the data given below, find the value of x when $y = 13.5$ using Lagrange's formula for inverse interpolation:

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.8	14.7	17.07	19.91

(6 marks)

c) i) Apply Gauss's forward central difference formula to estimate $f(32)$ from the following table:

x	25	30	35	40
y	0.2707	0.3027	0.3386	0.3794

(4 marks)

ii) If $\sqrt{12500} = 111.803399, \sqrt{12510} = 111.848111, \sqrt{12520} = 111.892805,$ $\sqrt{12530}$ $= 111.937483$, find $\sqrt{12516}$ by Gauss's backward formula. (4 marks)

QUESTION 4 (20 marks)

a) Find the gradient of the road at the middle point of elevation above a datum line of seven points of a road which are given below using Stirling's formula

x	0	300	600	900	1200	1500	1800
y	135	149	157	183	201	205	193

(5 marks)

b) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ using Romberg's method by taking $h = \frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$ hence obtain the approximate value of π

(9 marks)

a) Evaluate $I = \int_0^6 \frac{1}{1+x} dx$ using

i) Trapezoidal rule (2 marks)
ii) Simpsons' rule (4 marks)

QUESTION 5 (20 marks)

c) Find the two derivatives of $(x)^{1/3}$ at $x = 50$ and $x = 56$ given the table below:

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

(10 marks)

b) Convert the decimal number $(438)_{10}$ to a binary number (5 marks)

d) Find the value of $f'(0.5)$ using Stirling's formula from the following data

x	0.35	0.40	0.45	0.50	0.55	0.60	0.65
y	1.521	1.506	1.488	1.467	1.444	1.418	1.389

(5 marks)