(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **STA 346**

COURSE TITLE: QUALITY CONTROL AND ACCEPTANCE SAMPLING

DATE: 03/02/21 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION 1: (30 Marks) (COMPULSORY)

a) Give the three main objectives of a control chart **[3mks]**

b) What are the main applications of a control chart **[4mks]**

c) A large batch of items to be inspected using a single sampling scheme specified by the following values $n = 40$, $c = 2$, $\theta_1 = 0.02$, and $\theta_2 = 0.1$

  i. Define the operating characteristic of this sampling plan **[2mks]**

  ii. Find the probability of accepting a lot of quality $\theta = 0.05$ **[3mks]**

  iii. Find the consumer's risk and the producer's risk **[4mks]**

d) Workout the O.C curve and the ARL function for S^2-chart with upper warning limits given by $P[\sum(x_i - \bar{x})^2 > k] \leq 0.05$ and action is taken only if two consecutive values of S^2 fall beyond the upper warning limit (take $n = 12$, $\theta = \frac{\sigma^{2*}}{\sigma}$ and $\theta \to (-\infty, 0, \infty)$ **[6mks]**

e) suppose that the mean has shifted from μ to μ^* but σ^2 remain unchanged assuming normality (take $\alpha = 0.002$)

  i. Find the probability that the process is under control for the $\bar{x} - chart$ **[4mks]**

  ii. Show that the Average Run Length function of the $\bar{x} - chart$ is given by $\frac{1}{1-P(\theta)}$. Assuming that samples taken from the process are independent, where θ is the incoming quality **[4mks]**

QUESTION 2: (20 Marks)

a) Explain each of the following concepts

  i. Average sampling numbers [ASN] **[2mks]**

  ii. Average outgoing quality [AOQ] **[3mks]**

  iii. Lot tolerance percent defective (LTPD) **[3mks]**

  iv. Acceptance Quality Level [AQL] **[2mks]**

b) What do you understand by the moving average chart? Explain clearly how you can use it to determine whether a system is out of control or not. **[5mks]**

c) When do we use S^2- chart? Explain clearly how you can use it to determine whether a system is out of control or not. If $n = 4$ and $\alpha = 0.02$, obtain its upper action and warning limits. **[5mks]**

QUESTION 3: (20 Marks)

a) If n is large and p is moderately small and we let $\lambda = np$, obtain C-chart for the number of defectives per unit. (Take $\alpha = 0.001$ for action limit and $\alpha = 0.025$ for warning limit **[7mks]**

b) Obtain a single sampling for the proportion of defectives, fixing the producer's risk $\alpha = 0.09$ at $\theta_1 = 0.05$ and the consumer's risk $\beta = 0.1$ at $\theta_1 = 0.1$ and hence give your conclusion **[6mks]**

c) A large batch of items is to be tested by using double sampling inspection scheme specified by the following numbers $n_1 = 20,\ n_2 = 40,\ c_1 = 0,\ c_2 = c_3 = 2$

   i. Obtain an expression for the probability of accepting a batch in which the true proportion of defective is θ **[4mks]**

   ii. Obtain the value of this probability when $\theta = 0.05$ and $\theta = 0.1$ **[3mks]**

QUESTION 4: (20 Marks)

a) Briefly describe the important steps in constructing an $\bar{x} - chart$ **[5mks]**

b) Briefly compare the single sampling plan and the double sampling plan **[3mks]**

c) The data below are samples means and sample ranges for ten consecutive samples, each sample consisting of five measurements of a continuous random variable x. Assuming x is normally distributed plot $\bar{x} - control\ chart$ and comment on the degree of control

Sample No.	1	2	3	4	5	6	7	8	9	10
Sample mean	126.2	127.4	126.6	129.8	126.0	125.0	126.8	132.0	127.4	126.2
Sample Range	8	6	7	6	8	7	6	19	6	7

$a_n = 0.4299\ for\ n = 5$ **[6mks]**

d) Explain briefly how you use control chart for fractional defective (p −chart) to determine whether the process is in control or not and hence show its warning and action limit on a p-chart. (take $\alpha = 0.002$ for action limit and $\alpha = 0.05$ for warning limit) **[6mks]**

QUESTION 5: (20 Marks)

a) i. Construction a sequential sampling plan from a Bernoulli population that following values $\theta_0 = 0.02, \theta_1 = 0.08, \alpha = 0.05$ and $\beta = 0.1$ **[3mks]**

ii. An inspector test 40 units from a large lot. Would he have come to a decision to reject or accept the lot if he found the 10th , 18th , and 23rd unit defective. **[3mks]**

b) A company purchases large lots of items using a single sampling plan for which $n = 4$ and $c = 0$

i. Find the probability of acceptance of a lot in terms of proportion of defective items it contains. **[2mks]**

ii. What is the probability of

1. A lot containing 50% defective being accepted **[2mks]**
2. A lot containing 10% defective being rejected **[2mks]**

iii. Estimate the AQL (θ) corresponding to a producers risk of 5% and LTPD (θ) corresponding to consumer's risk of 10% **[3mks]**

iv. If rectification is agreed on, find the expression for the average outgoing quality (AOQ) in terms of the incoming quality. Find AOQ if $\theta = 0.05$ **[2mks]**

v. Calculate the average total inspection (ATI) of lots of size 100 of quality $\theta = 0.05$ **[3mks]**