(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: STA 443

COURSE TITLE: PROBABILITY AND MEASURE

DATE: 03/02/2021 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)(COMPULSORY)

(a) Define the following terms:

i. Measurable space (2 marks)

ii. Sigma-algebra (2 marks)

iii. Sample space (1 mark)

(b) Suppose that $A, B \in A$. Show that $\mu(B) = \mu(A \cap B) + \mu(B)$ (3 marks)

(c) Let $\{F_i \subset R^n : i \in N\}$ is countable collection of R^n. Show that

$$\mu^*(\cup_i^\infty F_i) \leq \sum_{i=1}^{\infty} \mu^*(F_i)$$

(5 marks)

(d) State and explain briefly two types of measure (4 marks)

(e) State and prove the Monotone Convergence Theorem (5 marks)

(f) Differentiate the following

i. Probability space and probability measure (2 marks)

ii. Measurable space and Measure space (2 marks)

(g) Suppose X and Y are independent random variables. Show that

$$E[X|Y = y] = E[X]$$

(4 marks)

QUESTION TWO (20 MARKS)

(a) What are Lebesgue measurable sets? (2 marks)

(b) Describe any two Lebesgue measurable sets (4 marks)

(c) If μ is a σ-finite measure on an algebra A of subsets of S. Show that:

i. there exists an increasing sequence (5 marks)

ii. there exists a disjoint σ-finite sequence (5 marks)

(d) Prove that if $0 \leq f_n \to f$ almost everywhere and $\int f_n d\mu \leq A < \infty$, then f is integrable and $\int f d\mu \leq A$ (4 marks)

QUESTION THREE (20 MARKS)

i. Let g_1 and g_2 be measurable functions on a common domain. Show that each set $\{\omega : g_1(\omega) < g_2(\omega)\}$, $\{\omega : g_1(\omega) = g_2(\omega)\}$ and $\{\omega : g_1(\omega) > g_2(\omega)\}$ is measurable (8 marks)

ii. Suppose $f = \sum_i x_i I_{Ai}$ is a non negative simple function, and $\{A_i\}$ decomposed from S into F sets, show that

$$\int f d\mu = \sum_i x_i \mu(A_i)$$

(6 marks)

iii. Let $P, q, r \in [1, \infty]$ satisfy $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$. Prove that for all measurable f and g defined on a space (X, A, μ), we haven $||fg||_r \leq ||f||_p ||g||_q$ (6 marks)

QUESTION FOUR (20 MARKS)

i. State and explain two properties of conditional expectation (4 marks)

ii. Let $X\sim\quad N(\mu,\sigma)$, obtain the characteristic function of X; hence use it to derive the mean and variance of X (10 marks)

iii. A random sample of 32 is taken from a population whose pdf is given by

$$f(x)=\begin{cases}\frac{1}{2\pi}x^{-\frac{1}{2}}e^{-\frac{x}{4}}\\ o,otherwise\end{cases}$$

Use CLT to compute the approximate probability that the mean of the random variable will extend 2.4 (6 marks)

QUESTION FIVE (20 MARKS)

i. State Fubini's theorem (2 marks)

ii. Find the integral $f(x,y) = x^2 + y^2$, on the domain $D = \{(x,y) \in R^2 : 0 < x < 1, x^2 < y < x\}$ (8 marks)

iii. If the sequence $\{B_n\}$ is of independent events and $\sum_n Pr\{B_n\} = \infty$. Show the probability that B_n occurs infinitely often is one. (10 marks)