



(Knowledge for Development)

## KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAI314

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATION

**DATE:** 03/02/2021 **TIME**: 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

**QUESTION ONE (30 MARKS)** 

- a) The slope at any point of a curve is 2x + 3y. If the curve passes through the origin, determine its equation. (5 Marks)
- b) Given that y = x is a solution of  $x \frac{d^2 y}{dx^2} + x \frac{dy}{dx} y = 0$  at  $x \ne 0$ . Find the general solution

of  $x\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x$  (4 Marks)

- c) Prove that  $L[Sin(at)] = \frac{a}{s^2 a^2}$  (4 Marks)
- d) By use of separable variables, solve first order differential equation (1-x)dy + (1-y)dx = 0 (4 Marks)
- e) Solve a homogeneous equation  $xdy ydx = \sqrt{x^2 + y^2} dx$ . (4 Marks)
- f) Solve a differential equation below which is reducible homogeneous form .

 $\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y + 5} \tag{6 Marks}$ 

- g) Define the following terms as used in ordinary differential equations. (3 Marks)
  - (i) Operator
  - (ii) Partial Differential Equation
  - (iii) Linear Differential Equation.

**QUESTION TWO (20 MARKS)** 

- a) Solve  $(D^3 D^2 6D)y = x^2 + 1$  (7 Marks)
- b) Apply partial differential method to solve  $z^2(p^2x^2 + x^2) = 1$  (8 Marks)
- c) Evaluate the functional  $I = \int_{0}^{1} \left[ y^{2} + \left( \frac{dy}{dx} \right)^{2} \right] dx$  by calculus variations along the paths (i)  $y = x^{2}$  (ii)  $y = (e^{x} 1)(e 1)$  (5 Marks)

**QUESTION THREE (20 MARKS)** 

a) Find the Laplace transforms of the following functions.

(i)  $f(x) = e^{ax}$  (5 Marks)

(ii) f(x) = Sinh(ax) (7 Marks)

b) Solve the function  $y = yp^2 + 2px$  given that the function is solvable for y only and

 $y = f(x, p), f(x, p, c) = 0, f(x, y, p) = 0, p = \frac{dy}{dx} = Q\left(x, p\frac{dp}{dx}\right)$  (5 Marks)

c) How long does it take for a given amount of money to double at 6% interest rate per annum compounded

(i) Annually. (1 Marks)

(ii) Continuously. (2 Marks)

**QUESTION FOUR (20 MARKS)** 

- a) Solve a homogeneous differential equation  $(D^3 + 1)y = Cos2x$  when Q(x) = bSin(ax)...or...bCos(nx) (10 Marks)
- b) Using Multiplier method, solve  $\frac{dx}{x(y^2 z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$  (10 Marks)

**QUESTION FIVE (20 MARKS)** 

- a) Compound Z is formed when two chemicals X and Y are combined. The resulting reaction between the two chemicals is such that each gram of X, 4g of Y is used. It is observed that 30 grams of compound Z is formed in 10 Minutes. Determine the amount of Z at any time if the rate of reaction is proportional to the amount of X and Y remaining when initially there were 50 grams of X and 32 grams of Y. How much compound Z is present after 15 Minutes. Interpret the solution as  $t \to \infty$  (10 Marks)
- b) Find the Inverse transforms of  $\frac{5s+2}{(s-2)^2+13}$  (4 Marks)
- c) In each of the following types of equation write two examples. (6 Marks)
  - (i) First Order Differential Equations
  - (ii) Second Degree Differential Equations
  - (iii) Ordinary Differential Equations