



*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2019/2020 ACADEMIC YEAR**  
**FOURTH YEAR SECOND SEMESTER**  
**SPECIAL/SUPPLEMENTARY EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF EDUCATION AND**  
**BACHELOR OF SCIENCE (MATHEMATICS)**

**COURSE CODE: MAT 436**

**COURSE TITLE: BIFURCATION AND DYNAMICS**

**DATE: 03/02/2021**

**TIME: 8 AM - 10 AM**

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

### QUESTION ONE (30 MARKS)

- a) Define the following (5 Marks)
- i) Bifurcation
  - ii) Stability
  - iii) critical point
  - iv) Autonomous system
  - v) Hyperbolic fixed point
- b) For a nonlinear system  $\dot{x} = f(x)$ , describe the following equilibrium points for this system
- i) Sink (1 mark)
  - ii) Source (1 mark)
  - iii) saddle (2 marks)
- c) Determine the equilibrium point for the following system (4 marks)
- $$\frac{dx_1}{dt} = x_2 + 1$$
- $$\frac{dx_2}{dt} = x_1 + x_2$$
- d) When do we say an Andronov Hopf bifurcation has occurred. (2 marks)
- e) State the stable manifold theorem. (3 marks)
- f) Consider the non linear system below
- $$\frac{dx_1}{dt} = x_1^2 - x_2^2 - 1$$
- $$\frac{dx_2}{dt} = 2x_2$$
- Classify all its critical points. (9 marks)
- g) State the Global stable manifold theorem. (3 marks)

### QUESTION TWO (20 MARKS)

Consider the nonlinear system below

$$\begin{aligned}\dot{x}_1 &= -x_1 \\ \dot{x}_2 &= -x_2 + x_1^2 \\ \dot{x}_3 &= x_3 + x_1^2\end{aligned}$$

Analyse the system by

- a) Finding the critical point. (4 marks)

b) Finding the eigen values at the critical point.

( 6 marks)

c) Show stability and unstability using the stable and unstable subspaces  $E^S$  and  $E^U$  of the linearized system above.

(10 marks)

### QUESTION THREE (20 MARKS)

Consider the following vector field on  $\mathbb{R}^2$

$$\dot{x} = x - y - x(x^2 + y^2)$$

$$\dot{y} = x + y - y(x^2 + y^2)$$

Construct a periodic Map from the system above. (20 marks)

### QUESTION FOUR (20 MARKS)

a) For the system  $\dot{x} = \mu - x^2$ , analyze and identify the type of Bifurcation. (10 marks)

b) Consider a nonlinear system  $\dot{x} = \mu x - x^2$ . Analyze the system and identify the type of Bifurcation. (10 marks)

### QUESTION FIVE (20 MARKS)

a) Find eigen values and corresponding eigen vectors of the matrix  $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$ . (10 marks)

b) Describe the trajectories through the points (1, 2) of the system. ( 10 marks)

$$\frac{dx}{dt} = \frac{x}{t}$$

$$\frac{dy}{dt} = y$$