(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAP 223/MAT206**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 17/02/2021 **TIME:** **8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 mks)

(a) Define the following terms (15mks)

(i) Greatest common divisor (2mks)

(ii) Group (3mks)

(iii) Subgroup (1mk)

(iv) Cyclic group (1mk)

(v) Abelian group (1mk)

(vi) Homomorphism (1mk)

(vii) Isomorphism (1mk)

(viii) Left coset and right coset (2mks)

(ix) A ring (3mks)

(b) Illustrate the principal of mathematical induction using the following statements

(i) $1+3+5+.... +(2n-1) = n^2$ for all $n \in \mathbb{N}$ (5mks)

(ii) Prove that $n^3 + 2n$ is divisible by 3 for all $n \in$ (5mks)

(c) Let (a, *) be a group for all $a,b,c \in G$, proof that $a*b = a*c$ implies $b=c$ for all $a,b,c \in G$, $b*a=c*a$ implies $b=c$ (5mks)

QUESTION TWO (20 mks)

(a) Let (G, *) be a group and $a, b \in G$. Proof that the equations $a*x=b$ and $y*a=b$ have unique solutions x and y in G. (7mks)

(b) Let (G, *) be a group and $a, b \in G$, proof that the inverse of $a*b$ is $(a*b)^{-1} = b^{-1}*a^{-1}$ (7mks)

(c) Let H be the subgroup generated by two elements a, b of a group G. proves that if ab=ba, then H is an abelian group. (6mks)

QUESTION THREE (20mks)

(a) Prove that in any group the orders of ab and ba are equal (6mks)

(b) Prove that the units in a commutative ring with unit elements form an abelian group (8mks)

(c) Show that if every element of the group G is its own inverse, then the group is abelian (3mks)

(d) Let H be the subgroup generated by two elements a, b of a group G. Prove that if ab=ba, then H is an abelian group. (3mks)

QUESTION FOUR (20 mks)

(a) If H is a subgroup of G and $a \in G$, let $aHa^{-1}= \{aha^{-1} \mid h \in H\}$.

(i) Show that aHa^{-1} is a subgroup of G (4mks)

(ii) If H is finite what is the order $o(aHa^{-1})$ (3mks)

(b) In a ring R if $x^3=x$ for all $x \in R$, show that R is commutative (13mks)

QUESTION FIVE (20mks)

(a) Let f: G → H be a homomorphism of groups. Denote the identity of G by e_G and the identity of H by e_H. Show that f

(i) Preserves identities: $f(e_G) = e_H$

(ii) Preserves inverses: for every $x \in G$, $f(x^{-1}) = f(x)$ (8mks)

(b) The center z of a group G is defined by $z = \{z \in G \mid zx = xz \text{ for all } x \in G\}$, prove that z is a subgroup of G. (4mks)

(c) Let G be the group of all non-zero complex numbers a+ib, (a,b real but not both zero) under multiplication and let

$H = \{a+ib \in G \mid a^2 + b^2 = 1\}$ verify that H is a subgroup of G. (6mks).