(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAP 223/MAT206

COURSE TITLE: ALGEBRAIC STRUCTURES II

DATE: 17/02/2021   **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 mks)

(a) Define the following terms (15mks)
- (i) Greatest common divisor (2mks)
- (ii) Group (3mks)
- (iii) Subgroup (1mk)
- (iv) Cyclic group (1mk)
- (v) Abelian group (1mk)
- (vi) Homomorphism (1mk)
- (vii) Isomorphism (1mk)
- (viii) Left coset and right coset (2mks)
- (ix) A ring (3mks)

(b) Illustrate the principal of mathematical induction using the following statements
- (i) $1+3+5+ \dots +(2n-1) = n^2$ for all $n \in \mathbb{N}$ (5mks)
- (ii) Prove that $n^3 + 2n$ is divisible by 3 for all $n \in$ (5mks)

(c) Let $(a, *)$ be a group for all $a,b,c \in G$, proof that $a* b = a * c$ implies b=c for all $a,b ,c \in G$, $b*a=c*a$ implies b=c (5mks)

QUESTION TWO (20 mks)

(a) Let $(G, *)$ be a group and $a, b \in G$. Proof that the equations $a* x=b$ and $y*a=b$ have unique solutions x and y in G. (7mks)

(b) Let $(G, *)$ be a group and $a, b \in G$, proof that the inverse of $a*b$ is $(a*b)^{-1} = b^{-1} *a^{-1}$ (7mks)

(c) Let H be the subgroup generated by two elements a, b of a group G. proves that if ab=ba, then H is an abelian group. (6mks)

QUESTION THREE (20mks)

(a) Prove that in any group the orders of ab and ba are equal (6mks)

(b) Prove that the units in a commutative ring with unit elements form an abelian group (8mks)

(c) Show that if every element of the group G is its own inverse, then the group is abelian (3mks)

(d) Let H be the subgroup generated by two elements a, b of a group G. Prove that if ab=ba, then H is an abelian group. (3mks)

QUESTION FOUR (20 mks)

(a) If H is a subgroup of G and $a \in G$, let $aHa^{-1}= \{aha^{-1} \mid h \in H\}$.
- (i) Show that aHa^{-1} is a subgroup of G (4mks)
- (ii) If H is finite what is the order $o(aHa^{-1})$ (3mks)

(b) In a ring R if $x^3=x$ for all $x \in R$, show that R is commutative (13mks)

QUESTION FIVE (20mks)

(a) Let $f: G \rightarrow H$ be a homomorphism of groups. Denote the identity of G by e_G and the identity of H by e_H. Show that f

    (i)   Preserves identities: $f(e_G) = e_H$

    (ii)   Preserves inverses: for every $x \in G$, $f(x^{-1}) = f(x)$ (8mks)

(b) The center z of a group G is defined by $z = \{z \in G \mid zx = xz \text{ for all } x \in G\}$, prove that z is a subgroup of G.   (4mks)

(c) Let G be the group of all non-zero complex numbers a+ib, (a,b real but not both zero) under multiplication and let

    $H = \{a+ib \in G \mid a^2 + b^2 = 1\}$ verify that H is a subgroup of G.   (6mks).