KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER
MAIN EXAMINATIONS

FOR THE DEGREE OFBSC (PHYSICS)

COURSE CODE: SPH 414

COURSE TITLE: QUANTUM MECHANICS II

DURATION: 2 HOURS

DATE: 12TH NOVEMBER, 2020 **TIME:9:00AM-12:00PM**

INSTRUCTIONS TO CANDIDATES

- Answer **QUESTION ONE** (Compulsory) and any other two (2) Questions.
- Indicate **answered questions** on the front cover.

Start every question on a new page and make sure question's number is written on each page

This paper consists of **3** printed pages. Please Turn Over

KIBU observes ZERO tolerance to examination cheating

The following rules of commutator algebra may be used where necessary

$[A,B] = AB - BA = -[B,A] = 0$

$[A,B+C] = [A,B] + [A,C]$

$[A+B,C] = [A,C] + [B,C]$

$[A,BC] = [A,B]C + B[A,C]$

$[AB,C] = A[B,C] + [A,C]B$

$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$

QUESTION ONE[30 Marks]

a) The un-normalized wave function for a particle moving in a one dimensional potential well V(x) is given by: $\psi(x) = exp\left(\frac{-ax^2}{2}\right)$. If the potential is to have minimum value at $x = 0$, determine (i) the eigenvalue (ii) the potential, $U(x)$. [6 Marks]

b) Simplify the arbitrary products of spin half operators; S_xS_y, S_zS_y, S_zS_x. [3 Marks]

c) Show that;

i) $J_+J_- = J^2 - J_z^2 + \hbar J_z$ [3 Marks] $S_xS_y = \frac{1}{2}i\hbar S_z$ [2 Marks]

ii) Find the commutators; $[\sigma_+\sigma_-]$ [2 Marks]

d) Verify that $S^2\alpha = \frac{3}{4}\hbar^2\alpha$ and $S_z\alpha = \frac{\hbar}{2}\beta$, where symbols have their usual meaning. [4 Marks]

e) Show that;
i) $[J_z, J_+] = J_+$ (iii) $[J_x, J_y] = iJ_z$ [4 Marks]

f) State the Pauli spin matrices $\sigma_x\sigma_y\sigma_z$ and show that $\sigma_x\sigma_y = -\sigma_y\sigma_x = i\sigma_z$ [6 Marks]

QUESTION TWO [20 Marks]

a) Write down an expression for the z-component of angular momentum, L_z, of a particle moving in the (x, y) plane in terms of its linear momentum components p_x and p_y. [2Marks]

b) Using the operator correspondence $P_x = -i\hbar\frac{\partial}{\partial x}$ etc., show that;

$$L_Z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

Hence show that $L_Z = -i\hbar\frac{\partial}{\partial\varphi}$, where the coordinates (x,y) and (r,φ)arerelated in the usual way. [4Marks]

c) Given that $L = r \times p$, show that $[L_{x,} L_{y,}] = i\hbar L_z$ [7Marks]

d) Use the rules of the commuter algebra to find the values of; $[L_{x,} p_y]$ and $[L^2{}_, L_x]$ [7Marks]

QUESTION THREE [20 Marks]

Obtain the angular momentum matrices for J^2, J_z, J_x and J_y for the case of j =1 particles. [20 Marks]

QUESTION FOUR [20 Marks]

a) What is perturbation theory? [2 Marks]

b) Show that the first order correction energy $E^{(1)}$ for a non degenerate level is just a perturbation H` averaged over the corresponding unperturbed state of the system. [12 Marks]

c) Given the unperturbed Hamiltonian for the linear harmonic oscillator $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2, k > 0$ and the unperturbed energy levels$E_n^{(0)} = \hbar\omega\left(n + \frac{1}{2}\right)$ n=0, 1, 2 …, Write the perturbed eigen functions and eigen values [6 Marks]

QUESTION FIVE[20 Marks]

a) The one-dimensional time-independent Schrodinger equation is;

$$\left(-\frac{\hbar^2}{2m}\right)\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Give the meanings of the symbols in this equation. [4 Marks]

b) A particle of mass m is contained in a one-dimensional box of width a.The potential energyU(x) is infinite at the walls of the box (x =0 andx=a) and zero in between (0<x<a).Solve the Schrodinger equation for this particle and hence show that thenormalized solutions have the form;

$$\psi_n(x) = \left(\frac{2}{a}\right)^{0.5} sin\left(\frac{n\pi x}{a}\right)$$

With energy;

$$E_n = \frac{\mathrm{h}^2 n^2}{8ma^2}$$

where nis an integer (n>0). [10 Marks]

c) For the casen=3, find the probability that the particle will be located inthe region a/3 <x<2a/3. [3Marks]

d) Sketch the wave-functions and the corresponding probability density distributionsfor the casesn=1,2 and 3. [3Marks]

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