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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
SPECIAL/ SUPPLEMENTARY EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAA 211/MAT 203/MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: 16/02/2021

TIME: 11 AM - 1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question 1: Compulsory (30 marks)

- a. Find the terminal point of the vector $v = 7i - j + 3k$, given that the initial point is $P = (-2, 3, 5)$ (3mks)
- b. When do we say two vectors are orthogonal? (2mks)
- c. Given the vectors $u = 3i - j + 2k$ and $v = -4i + 2k$ find the angle between u and v (3mks)
- d. Show that the diagonals of a parallelogram bisect each other (6 mks)
- e. Prove that the dot product of two vectors $u \cdot v = |u||v| \cos \theta$ where θ is the angle between the vectors (4mks)
- f. If $f = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$ find
i). curl f ii). div f iii). div(curl f) at $(1, -1, 1)$ (5mks)
- g. The position of a moving particle is given by
 $r(t) = 2 \cos t i + 2 \sin t j + 3t k$. Find the vectors T, N, B and the curvature (7mks)

Question 2

- a. When do we say a vector is irrotational? Prove that that the vector $F = (y + yz)i + (x + 3z^3 + xz)j + (9yz^2 + xy - 1)k$ is a irrotational hence find its potential function ϕ for F (10mks)
- b. Evaluate $\int_c xy^2 dx$ on the quarter circle c defined by $x = \cos t, y = 4 \sin t$ (10mks)

Question 3

- a. Find the work done by $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curve $r(t) = ti + t^2j + t^3k$ from $(0, 0, 0)$ to $(1, 1, 1)$ (10mks)
- b. State the divergence theorem hence evaluate $\iint_s F \cdot nds$ where the vector $F = 4xzi - y^2j + yzk$ and s is the surface of a cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (10mks)

Question 4

- a. Verify Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$ where c is the region bounded by the curves $y = x$ and $y = x^2$ (10mks)
- b. Define the following terms (10mks)
- The unit tangent \mathbf{T}
 - The principal normal \mathbf{N}
 - The bi-normal \mathbf{B}
 - The curvature, k
 - Torsion

Question 5

- a. Verify Stokes' theorem for $F = xyi + yzj + xzk$ where s is that part of the cylinder $z = 1 - x^2$ for $0 \leq x \leq 1, -2 \leq y \leq 2$ assuming s is oriented upward (10mks)
- b. Let $F = x^2 + y^2 + z^2$; evaluate $\iiint_v Fdv$ where v is the region bounded by $x + y + z = a$ ($a > 0$), $x = 0, y = 0, z = 0$ (10mks)