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*(Knowledge for Development)*

**KIBABII UNIVERSITY**  
**UNIVERSITY EXAMINATIONS**  
**2019/2020 ACADEMIC YEAR**  
**SECOND YEAR SECOND SEMESTER**  
**SPECIAL/ SUPPLEMENTARY EXAMINATION**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE**

**COURSE CODE:** MAA 211/MAT 203/MAT 214

**COURSE TITLE:** VECTOR ANALYSIS

**DATE:** 16/02/2021

**TIME:** 11 AM - 1 PM

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INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

**Question 1: Compulsory (30 marks)**

- a. Find the terminal point of the vector  $v = 7i - j + 3k$ , given that the initial point is  $P = (-2, 3, 5)$  (3mks)
- b. When do we say two vectors are orthogonal? (2mks)
- c. Given the vectors  $u = 3i - j + 2k$  and  $v = -4i + 2k$  find the angle between  $u$  and  $v$  (3mks)
- d. Show that the diagonals of a parallelogram bisect each other (6 mks)
- e. Prove that the dot product of two vectors  $u \cdot v = |u||v| \cos \theta$  where  $\theta$  is the angle between the vectors (4mks)
- f. If  $f = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$  find  
i). curl  $f$  ii). div  $f$  iii). div(curl  $f$ ) at  $(1, -1, 1)$  (5mks)
- g. The position of a moving particle is given by  
 $r(t) = 2 \cos t i + 2 \sin t j + 3t k$ . Find the vectors  $T, N, B$  and the curvature (7mks)

**Question 2**

- a. When do we say a vector is irrotational? Prove that that the vector  $F = (y + yz)i + (x + 3z^3 + xz)j + (9yz^2 + xy - 1)k$  is a irrotational hence find its potential function  $\phi$  for  $F$  (10mks)
- b. Evaluate  $\int_c xy^2 dx$  on the quarter circle  $c$  defined by  $x = \cos t, y = 4 \sin t$  (10mks)

### Question 3

- a. Find the work done by  $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$  over the curve  $r(t) = ti + t^2j + t^3k$  from  $(0, 0, 0)$  to  $(1, 1, 1)$  (10mks)
- b. State the divergence theorem hence evaluate  $\iint_s F \cdot nds$  where the vector  $F = 4xzi - y^2j + yzk$  and  $s$  is the surface of a cube bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$  (10mks)

### Question 4

- a. Verify Green's theorem for  $\oint_c (xy + y^2)dx + x^2dy$  where  $c$  is the region bounded by the curves  $y = x$  and  $y = x^2$  (10mks)
- b. Define the following terms (10mks)
- The unit tangent  $\mathbf{T}$
  - The principal normal  $\mathbf{N}$
  - The bi-normal  $\mathbf{B}$
  - The curvature,  $k$
  - Torsion

### Question 5

- a. Verify Stokes' theorem for  $F = xyi + yzj + xzk$  where  $s$  is that part of the cylinder  $z = 1 - x^2$  for  $0 \leq x \leq 1, -2 \leq y \leq 2$  assuming  $s$  is oriented upward (10mks)
- b. Let  $F = x^2 + y^2 + z^2$ ; evaluate  $\iiint_v Fdv$  where  $v$  is the region bounded by  $x + y + z = a$  ( $a > 0$ ),  $x = 0, y = 0, z = 0$  (10mks)