



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2019/2020 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER
MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN
MATHEMATICS

COURSE CODE: MAT 869/MAT 817

COURSE TITLE: COMPLEX ANALYSIS

DATE: 12/02/2021

TIME: 2 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question1 [20 marks]

(a) (i) Express $(1+i)^{1/6}$ in rational Cartesian form. [5 marks]

(ii) Show that $e^{ix} + e^{-ix} = 2 \cos x$, $e^{ix} - e^{-ix} = 2i \sin x$ [6 marks]

(b) Let D be a rectangular region bounded by lines $x=0, y=0, x=2$ and $y=1$.

Define the mapping $\omega(z) = (12+i)z + (1+2i)$ on D into D' . [2 marks]

(i) Show that ω is a conformal mapping. [2 marks]

(ii) Obtain the translation, rotation and dilation factor, of D into D' [2 marks]

(c) Determine the residues of $f(z) = \frac{z}{(z^2 + 144)^2}$ [3 marks]

Question 2 [20 marks]

(a) (i) Discuss the existence of $f'(z_0)$, the derivative of $f(z)$ on the complex plane [5marks]

ii) If $f(z) = z^2 \bar{z}$, find $f'(z)$ the derivative of $f(z)$ [7marks]

(b) Evaluate the integral : $\oint_{|z|=3} \frac{z}{(z^2 - 9)^3} dz$ [5 marks]

(c) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. [3marks]

Question 3 [20 marks]

(a) If $f(z) = z\bar{z}$ find $\lim_{z \rightarrow z_0} \left\{ \frac{f(z) - f(z_0)}{z - z_0} \right\}$. Discuss the existence of $f'(z_0)$, the derivative of $f(z)$ on the complex plane. [6marks]

(b) Find all the points at which the function $f(z) = x^2 - y^2 + x + i(2xy - y)$ is analytic. [6 marks]

(c) Evaluate the integral : $\oint_{|z|=3} \frac{z}{(z^2-9)^3} dz$

[8marks]

Question 4 [20 marks]

(a) Given $f(z) = \frac{z+11}{(z+1)^3(z^2+1)^2}$, identify the poles of $f(z)$

[5marks]

(b) Let $f(z) = e^{z^2} : z = x+iy$

Discuss exhaustively and explicitly the differentiability of $f(z)$

[15marks]

Question 5 [20 marks]

(a) Evaluate the integral $\int_C z^2 dz : C$ is the curve $y = \frac{1}{x^2}$ from $z = 1+i$ to $z = 3 + \frac{i}{19}$ [3marks]

(b) Suppose that a function f is analytic in a star D . Suppose further that C is a closed contour lying in D . Prove that $\oint_C f(z) dz = 0$. [8marks]

(c) Determine the value of the contour integral $\oint_{|z|=13} \frac{e^z + \sin z}{(z^2-25)(z^2-49)} dz$ where the contour of integration is the circle centre at 0 and radius 3 followed in the positive (anticlockwise) direction. [9marks]