(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS

COURSE CODE: MAT 869/MAT 817

COURSE TITLE: COMPLEX ANALYSIS

DATE: 12/02/2021 **TIME:** 2 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Any THREE Questions

TIME: 3 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question1 [20 marks]

(a) (i) Express $(1+i)^{1/6}$ in rational Cartesian form. [5 marks]

(ii) Show that $e^{ix}+e^{-ix}=2\cos x$, $e^{ix}-e^{-ix}=2i\sin x$ [6 marks]

(b) .Let D be a rectangular region bounded by lines $x=0, y=0, x=2$ and $y=1$.

Define the mapping $\omega(z)=(12+i)z+(1+2i)$ on D into D'. [2 marks]

(i) Show that ω is a conformal mapping. [2 marks]

(ii) Obtain the translation , rotation and dilation factor , of D into D' [2 marks]

(c)Determine the residues of $f(z)=\dfrac{z}{(z^2+144)^2}$ [3 marks]

Question 2 [20 marks]

(a)i) Discuss the existence of $f'(z_0)$, the derivative of $f(z_0)$ on the complex plane [5marks]

ii) If $f(z)=z^2\bar{z}$, find $f'(z)$ the derivative of $f(z)$. [7marks]

(b) Evaluate the integral : $\oint_{|z|=3}\dfrac{z}{(z^2-9)^3}dz$ [5 marks]

(c) Prove that $u=e^{-x}(x\sin y-y\cos y)$ is harmonic. [3marks]

Question 3 [20 marks]

(a) If $f(z)=z\bar{z}$ find $\lim\limits_{z\to z_0}\left\{\dfrac{f(z)-f(z_0)}{z-z_0}\right\}$. Discuss the existence of $f'(z_0)$, the derivative of $f(z)$ on the complex plane. [6marks]

(b) Find all the points at which the function $f(z)=x^2-y^2+x+i(2xy-y)$ is analytic. [6 marks]

(c) Evaluate the integral : $\oint_{|z|=3} \frac{z}{\left(z^2-9\right)^3} dz$ [8marks]

Question 4 [20 marks]

(a) Given $f(z)=\frac{z+11}{(z+1)^3\left(z^2+1\right)^2}$, identify the poles of $f(z)$ [5marks]

(b)Let $f(z)=e^{z^2}$ $:z=x+iy$

Discuss exhaustively and explicitly the differentiability of $f(z)$ [15marks]

Question 5 [20 marks]

(a) Evaluate the integral $\oint_C z^2 dz : C$ *is the curve* $y=\frac{1}{x^2}$ *from* $z=1+i$ *to* $z=3+\frac{i}{19}$ [3marks]

(b) Suppose that a function f is analytic in a star D. Suppose further that C is a closed contour lying in D. Prove that $\oint_C f(z)dz=0$. [8marks]

(c) Determine the value of the contour integral $\oint_{|z|=13} \frac{e^z+\sin z}{\left(z^2-25\right)\left(z^2-49\right)} dz$ where the contour of integration is the circle centre at 0 and radius 3 followed in the positive (anticlockwise) direction. [9marks]