(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 403**

COURSE TITLE: **COMPLEX ANALYSIS II**

DATE: 04/02/2021 **TIME:** **2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

UNIT CODE: MAT 403 **UNIT TITLE:** COMPLEX ANALYSIS II SPECIAL EXAM

INSTRUCTIONS: Answer question one and any other two

QUESTION ONE (Compulsory)

a) Define a Harmonic function and hence show that the function $\emptyset = x^3 - 3xy^2 + 2y$ can be a real part of analytic function. Find the imaginary part of the analytic function. (10 marks)

b) Show that $\oint_c \frac{\sin z}{z^4} dz = -\frac{\pi}{3} i$, where $c: \ |z| = 1$,described in a positive direction. (5 marks)

c) Discuss the singularity of the following function: $f(z) = \frac{z \cos z}{(z-1)(z^2+1)^2(z^2+3z+2)}$ (7 marks)

d) Define the following terms; Laurent series, principal and analytic part of a Laurent series, pole of order N, singularity, isolated singularity and non-isolated singularity. (8 marks)

QUESTION TWO

a) Define the residue of a function $f(z)$ and illustrate the relationship between the residue and the Laurent series of the function $f(z)$. (6 marks)

b) Expand the function $f(z) = \frac{1}{(z+1)(z+2)}$ in a Laurent series in the powers of $(z-1)$ valid in the annular domain containing the point $z = \frac{7}{2}$. State the domain in which the series converges to $f(z)$. (14 marks)

QUESTION THREE

a) Using residues, show that $\int_{-\infty}^{\infty} \frac{x^2+3}{(x^2+1)(x^2+4)} dx = \frac{5}{6}\pi$ (10 marks)

b) Find a Schwartz-Christoffel transformation that maps the upper half plane H to the inside of a triangle vertices $-1, 0$ and i. (10 marks)

QUESTION FOUR

a) Use the function $f(z) = \frac{1}{2}ln(x^2 + y^2) + i \arg z$ to show that if a function $f(z) = u(x, y) + i\, v(x, y)$ is an analytic function and $c_1, c_2, c_3, \dots \& k_1, k_2, k_3, \dots$ are real constants. Then the family of curves in the $xy-plane$ along which $u = c_1, u = c_2, \dots$ is orthogonal to the family given by $v = k_1, k_2, \dots$

(10 marks)

b) Find the residue of the following functions;

$f(z) = \frac{4-3z}{z^2-z}$, $f(z) = \frac{e^z}{(z^2+1)z^2}$, $f(z) = \frac{\sin z}{(z^2+z+1)\cos z}$ (10 marks)

QUESTION FIVE

a) Find $I = \int_0^{2\pi} \frac{\cos 2\theta d\theta}{5-4\sin\theta}$ (10 marks)

b) Consider the contour C defined by $x = y, x > 0$ and the contour C_1 defined by $x = 1, y \geq 1$. Maps these two curves using $w = \frac{1}{z}$ and verify that their angle of intersection is preserved in size and direction. (10 marks)