(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **MAT 407**

COURSE TITLE: **FUNCTIONAL ANALYSIS**

DATE: 03/02/2021 **TIME:** **8 AM -10 AM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (COMPULSORY)(30 marks)

a) Define the nested sequence of a subspace $X \subseteq \mathbb{R}$ **(3 marks)**

b) Show that a set A is bounded if and only if the $diamA < \infty$ i.e diameter of A is finite.

(6 marks)

c) Define the term contraction mapping **(3 marks)**

d) If (X, d) is a metric space and $E \subseteq X$, then show that E is open if E^c is closed **(4marks)**

e) Define the term metric space of a set X with a metric d. **(6 marks)**

f) Let (X, d) be a complete metric space $f: (X, d) \to (X, d)$ a function, for some $p \in N$ and > 0, for a contraction map f^p, Show that f has a unique fixed point. **(8 marks)**

QUESTION TWO (20marks)

a) Let (X, d) be a metric space, Prove that

i) $\emptyset$, X are open in (X, d) **(6 marks)**

ii) Finite intersection of open sets is open in (X, d) **(8 marks)**

b) if E is a compact subset of (X, d) then show that E is bounded **(6 marks)**

QUESTION THREE(20 marks)

Let (X, d) be a metric space, then the following conditions are equivalent

i) (X , d) is a complete . **(6 marks)**

ii) For every nested sequence (A_n) of closed subsets of X with $diamA_n \to 0$ as $n \to \infty$

We have

$\bigcap_{n=1}^{\infty} A_n = \{x\}$ for some $x \in X$. Prove this theorem. **(12 marks)**

And

name the theorem you have just proved. **(2 marks)**

QUESTION FOUR (20 marks)

a). Define the open neighborhood of a point in a metric space(X, d) **(3 marks)**

b). Let(X, d) be a metric space and Y be a nonvoid subset of X .

Let $dy(x, y) = \mathrm{d}(x, y)$ $\forall$ x, y $\in$ Y , and then show that (Y, dy) is a metric space.

(9 marks)

c). State without proof the Banach Fixed Point Theorem. **(4 marks)**

d). Let (X, d) be a metric space. Then show that any convergent sequence(x_n) in (X, d) is Cauchy. **(4 marks)**

QUESTION FIVE(20 marks)

a) Show that the metric space $[C[0,1], d_\infty]$ is complete. **(10 marks)**

b) Let (X, d) is a metric space and(Y, d_y) a subspace of (X, d) . Then prove that if $E \subseteq Y$ then, E is open in (Y, d_y) iff $E = G \cap Y$ for some subset G open in (X, d) .

(10 marks)