(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: MAT 423

COURSE TITLE: ODE II

DATE: 05/02/2021 **TIME:** 11 AM -1 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION ONE (30mks)

a) Show that the solutions $\emptyset_1(x) = e^x, \emptyset_2(x) = e^{-2x}$ and $\emptyset_3(x) = e^{-x}$ to a differential equation are linearly independent. (8mks)

b) Show that the solution of the system

$\dot{x} = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -1 & 2 \\ -3 & -2 & -1 \end{pmatrix} x$ is asymptotically stable. (10mks)

c) If $x(t) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$ is a general Solution ,

Find a particular solution given that $x(o) = I$ (12mks)

QUESTION TWO (20mks)

If $y = \sum_{n=1}^{\infty} \alpha_n x^{n+p}$, apply the appropriate differentiation in the Bessel's equation given by:

$x^2y + xy' + (x^2 - p^2)y = 0$ to show that $a_n = -\frac{a_{n-2}}{n(2p-n)}$ (10mks)

and that $a_4 = -\frac{a_n}{4(2p+4)} = \frac{a_0}{2.4(2p+2)(2p+4)}$ (10mks)

QUESTION THREE (20mks)

a) Use Picard's method to approximate the value of y when $x = 0.1$ given that $y = 1$ when $x = 0$ and $\frac{dy}{dx} = 3x + y^2$ (8mks)

b) Solve the system of linear equations given by:

$$(D-1)x + Dy = 2t + 1$$

$$(2D+1)x + 2Dy = t$$ (12mks)

QUESTION FOUR (20mks)

Find the fundamental matrix for the system of equation $X' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x$ (20mks)

QUESTION FIVE (20mks)

a) Explain what is meant by stability of linear systems and hence give the conditions for a solution $\boldsymbol{x} = \emptyset(x)$ of $\dot{\boldsymbol{x}} = A\boldsymbol{x}$ to be

i. Stable (2mks)

ii. Unstable (1mk)

iii. asymptotically stable (1mk)

b) Determine the stability or instability of the following systems of differential equations;

i. $\dot{\boldsymbol{x}} = \begin{pmatrix} 1 & 5 \\ 5 & 1 \end{pmatrix} \boldsymbol{x}$ (4mks)

ii. $\dot{\boldsymbol{x}} = \begin{pmatrix} 0 & -3 \\ 2 & 0 \end{pmatrix} \boldsymbol{x}$ (3mks)

iii. Solve the system of linear differential equations given by:

$$2\frac{dx}{dt} + \frac{dy}{dt} - 4x - y = e^t$$

$$\frac{dx}{dt} + 3x + y = 0$$ (9mks)