(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

AND BACHELOR OF EDUCATION

COURSE CODE: MAT 401

COURSE TITLE: TOPOLOGY I

DATE: 11/02/2021 **TIME:** 1 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) Define the following terms: limit point, interior point, closed set, boundary point and adherent point. (5 marks)

b) Show that the intersection $\tau_1 \cap \tau_2$ of any two topologies τ_1 and τ_2 on X is also a topology on X (5 marks)

c) Consider the topology $\tau_1 = \{X, \emptyset, \{a\} \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ on $X = \{a,b,c,d,e\}$. Determine B', the derived set of $B = \{b\} \subset X$. (6 marks)

d) Define a homeomorphism. (2 marks)

e) Let $X = \{1,2,3\}$. Show that $\beta = \{\{1,2\}, \{2,3\}\}$ cannot be a base for any topology X. (6 marks)

f) If $A \subset B$, then $\bar{A} \subset \bar{B}$. Prove. (6 marks)

QUESTION 2 (20 MARKS)

a) Define a topological space. (3 marks)

b) The class $\tau = \{X, \emptyset, \{a\} \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ is a topology on $X = \{a,b,c,d,e\}$.

   i. List the closed subsets of X (2 marks)

   ii. Determine the closure of the sets $\{a\}$, $\{b\}$ and $\{c,e\}$. (6 marks)

   iii. Which sets in (ii) are dense in X? (1 mark)

c) Prove that if $A \subset B$, then every limit point of A is a limit point B. (8 marks)

QUESTION 3 (20 MARKS)

a) Define a Hausdorff space. (3 marks)

b) Prove that all metric spaces are Hausdorff spaces. (7 marks)

c) Let $\tau = \{X, \emptyset, \{a\} \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ be a topology on $X = \{a,b,c,d,e\}$. Find the neighbourhood system of:

   i. The point e (2 marks)

   ii. The point c (2 marks)

d) A set G is open if and only if it is a neighbourhood of each of its points. Prove. (6 marks)

QUESTION 4 (20 MARKS)

a) Define continuity of a function between topological spaces. (2 marks)

b) Let $X; Y; Z$ be topological spaces, and let $f: X \to Y$ and $g: Y \to Z$ be continuous functions. Prove that the composition $g \circ f: X \to Z$ of the functions f and g is continuous. (9 marks)

c) Let $\{\tau_i\}$ be a collection of topologies on a set X. If a function $f: X \to Y$ is continuous with respect to each τ_i, prove that f is continuous with respect to the intersection topology $\tau = \cap_i \tau_i$. (9 marks)

QUESTION 5 (20 MARKS)

a) Let $\tau_1 = \{X, \emptyset, \{a\} \{a,b\}, \{a,c,d\}, \{a,b,c,d\}, \{a,b,e\}\}$ be a topology on $X = \{a,b,c,d,e\}$. Let $A = \{a,b,c\} \subset X$. Find

   i. $Int\,(A)$, the interior of A. (4 marks)

   ii. $Ent\,(A)$, the exterior of A. (4 marks)

   iii. $\partial(A)$, the boundary of A. (4 marks)

b) Let A be a subset of a topological space X and $\bar{A}$ be the closure of A. Show that $\bar{A} = Int\,(A) \cup \partial\,(A)$. (8 marks)