(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE MATHEMATICS

COURSE CODE: MAT 100

COURSE TITLE: MATHEMATICS FOR TECHNOLOGISTS

DATE: 16/02/2021 **TIME:** 8 AM - 10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

a) Define the following terms
   i. Population
   ii. Complement sets
   iii. Geometric progression (3mks)

b) Show that $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ has complex eigenvalues (3mks)

c) At Dan's automatic shop 50 cars were inspected. 23 Needed new brakes, 34 needed new exhaust systems and 6 needed neither repair.
   i. How many needed both repairs
   ii. How many needed new brakes but not new exhaust system. (6mks)

d) Evaluate $\lim\limits_{x \to 1} \frac{x^2+x-2}{x^2-3x+2}$ (2mks)

e) Prove that $\lim\limits_{x \to 3}(2x+1) = 7$ (3mks)

f) Obtain the derivative of the function $y = \sqrt{x}$ using the delta method. (4mks)

g) Differentiate (4mks)
   i. $y = x(x+3)^4$
   ii. $y = \cos(x^3 - 1)$

h) Find the slope of the line tangent to its graph at the point (2,2) of the equation (3mks)
$$y^2 + x^2y = 3x^2$$

i) Find $\frac{d^2y}{dx^2}$ given that $y = e^{-3x}cos4x$ (2mks)

QUESTION TWO (20MARKS)

a) Proof the DeMorgan's theorems
   i. $(A \cup B)^1 = A^1 \cap B^1$
   ii. $(A \cap B)^1 = A^1 \cup B^1$ (10mks)

b) If $U = \{a, b, c, d, f, g, h, i, j, k\}$ $X = \{a, c, g, i, k\}$ and $y = \{a, f, h, j, k\}$
Proof (a) above (10mks)

QUESTION THREE (20MARKS)

a) Define a matrix (1mks)

b) If $A = \begin{Bmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \\ 1 & 3 & 2 \end{Bmatrix}$ $B = \begin{Bmatrix} 2 & 4 & 1 \\ 1 & 2 & 0 \\ 5 & 9 & 6 \end{Bmatrix}$ and $C = \begin{Bmatrix} 8 & 0 & 2 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{Bmatrix}$

Find $\frac{1}{2}A + 3B - 2A$ (3mks)

c) Compute the rank of A (6mks)

$$A = \begin{pmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & -5 & 8 \end{pmatrix}$$

d) Find eigenvalues and eigenvectors of the matrix (10mks)

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

QUESTION FOUR (20MARKS)

a) The third term geometric sequence is 3 and the fifth term is $\frac{3}{4}$. Write down the first four terms of the sequence. (6mks)

b) In an arithmetic progression, the thirteenth term is 27 and the seventh term is three times the second term. Find the first term, common difference and the sum of the first ten terms. (7mks)

c) A geometric progression has positive terms. The sum of the first six terms is nine times the sum of the first three terms. The seventh term is 320. Find the common ratio and the first term. Find the smallest value of n such that the sum to first n terms of the progression exceeds 10^6. (7mks)

QUESTION FIVE (20MARKS)

a) In a group of 100 customers at a big pizza, 80 of them ordered mushrooms on thir pizza, 72 ordered pepperoni and 60 ordered both mushroom and pepperoni. (6mks)

i. How many ordered mushroom and no pepperoni
ii. How many ordered pepperoni but no mushroom
iii. How many ordered neither of these two toppings

b) A survey of 85 students asked them about the subject they liked to study. 35 liked math, 37 liked history and 26 physics. 20 liked math/history, 14 math/pysics and 3 history/physics. 2 liked the three subjects. (6mks)

i. How many like math or physics
ii. How many didn't like the three subjects
iii. How many like math/history but not physics

c) Differentiate the following and give examples in each case (8mks)

i. Set union and set intersection.
ii. Equivalent and equal set