(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 434**

COURSE TITLE: DIFFERENTIAL GEOMETRY

DATE: 10/11/2020 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

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QUESTION ONE (30marks)

a). Define the following terms

i). Rectifiable arc (2 mks)

ii). Curvature of a curve (1 mk)

iii). Bertrand curves (1 mk)

iv). Ordinary point on a surface (2 mks)

b). Determine the curvature of the curve $\boldsymbol{r}(t) = t\boldsymbol{i} + 3\sin t\,\boldsymbol{j} + 3\cos t\,\boldsymbol{k}$. (4 mks)

c). Let γ be a curve lying on the surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ where $u = u(t), v = v(t), a \leq t \leq b$. Prove that the length of the arc on the curve is given by $\int_a^b \sqrt{I}\ dt$ where I is the first fundamental form of a surface. (4 mks)

d). Find the volume of the parallelepiped with vertices at O,P,Q and R having coordinates $(0,0,0)$, $(1,2,4)$, $(-2,3,-5)$ and $(0,1,-1)$ respectively. (3 mks)

e). Determine the arc length of the curve $\boldsymbol{X}(t) = 4e^{2t}\cos t\,\hat{e}_1 + 4e^{2t}\sin t\,\hat{e}_2 + 4e^{2t}\,\hat{e}_3$ for $0 \leq t \leq \frac{\pi}{2}$. (5 mks)

f). Find the equation of the tangent line and normal plane to the curve $\boldsymbol{X}(t) = (1+t)\hat{e}_1 - t^2\hat{e}_2 + (1+t^3)\hat{e}_3$ at $t = 1$. (4 mks)

g). State and derive the first fundamental form of a surface $\boldsymbol{X} = \boldsymbol{X}(u, v)$ whose class is more or equal to 1. (4 mks)

QUESTION TWO (20marks)

a). Find the unit tangent vector to the curve $\boldsymbol{X}(t) = \langle 2t + t^2, t + \frac{t^2}{2}, 2t^2 \rangle$ at $t = 1$. (3 mks)

b). Prove that the curvature of the curve $\boldsymbol{X} = \boldsymbol{X}(t)$ is

$$\kappa = \frac{|X' \times X''|}{|X'|^3}$$ (5 mks)

c). Determine the lines of curvature to the helicoid $r(u, v) = \langle u\cos v\,, u\sin v\,, av \rangle$. (12 mks)