(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 436**

COURSE TITLE: **BIFURCATION ANDDYNAMICS**

DATE: 11/11/2020 **TIME: 2 PM - 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following (5 Marks)

   i) Bifurcation
   ii) Stability
   iii) critical point
   iv) Autonomous system
   v) Hyperbolic fixed point

b) For a nonlinear system $\dot{x} = f(x)$, describe the following equilibrium points for this system

   i) Sink (1 mark)
   ii) Source (1 mark)
   iii) saddle (2 marks)

c) Determine the equilibrium point for the following system (4 marks)

$$\frac{dx_1}{dt} = x_2 + 1$$

$$\frac{dx_1}{dt} = x_1 + x_2$$

d) When to we say an Andronov Hopf bifurcation has occurred. (2 marks)

e) State the stable manifold theorem. (3 marks)

f) Consider the non linear system below

$$\frac{dx_1}{dt} = x_1^2 - x_2^2 - 1$$

$$\frac{dx_2}{dt} = 2x_2$$

   Classify all its critical points. (9 marks)

g) State the Global stable manifold theorem. (3 marks)

QUESTION TWO (20 MARKS)

Consider the nonlinear system below

$$\dot{x}_1 = -x_1$$

$$\dot{x}_2 = -x_2 + x_1^2$$

$$\dot{x}_3 = x_3 + x_1^2$$

Analyse the system by

a) Finding the critical point. (4 marks)

b) Finding the eigen values at the critical point. (6 marks)

c) Show stability and unstability using the stable and unstable subspaces E^S and E^U of the linearized system a bove. (10 marks)

QUESTION THREE (20 MARKS)

Consider the following vector field on $\Re^2$

$$\dot{x} = x - y - x(x^2 + y^2)$$
$$\dot{y} = x + y - y(x^2 + y^2)$$

Construct a periodic Map from the system above. (20 marks)

QUESTION FOUR (20 MARKS)

a) Find eigen values and corresponding eigen vectors of the matrix $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$. (10 marks)

b) Describe the trajectories through the points (1, 2) of the system . (10 marks)

$$\frac{dx}{dt} = \frac{x}{t}$$
$$\frac{dy}{dt} = y$$

QUESTION FIVE (20 MARKS)

a) For the system $\dot{x} = \mu - x^2$, analyse and identify the type of Bifurcation. (10 marks)

b) Consider a nonlinear system $\dot{x} = \mu x - x^2$. Analyze the system and identify the type of Bifurcation. (10 marks)