(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **STA 442**

COURSE TITLE: MULTIVARIATE ANALYSIS

DATE: 10/11/2020 **TIME: 2 PM - 4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) The following are data for young boys in a given region

Individual	Height (cm)	Chest circumference (cm)	MUAC (cm)
1	78	60.6	16.0
2	76	58.1	12.5
3	92	63.2	14.4
4	81	59.0	14.0
5	81	60.8	15.5
6	84	59.5	14.0

MUAC-Mid-upper-arm circumference

Obtain the sample variance-covariance matrix and comment on the relationship between the different variables. (13 marks)

b) Find the maximum likelihood estimators of the mean vector $\underline{\mu}$ and covariance matrix Σ based on the data matrix (6 marks)

$$x = \begin{bmatrix} 4 & 1 \\ -1 & 3 \\ 3 & 5 \end{bmatrix}$$

c) Let $\underline{x} = [0,4,3]$ and $\underline{y} = [-1,2,0]$. Find

(i) The length of $\underline{x}$ (2marks)

(ii) The angle between $\underline{x}$ and $\underline{y}$ (3marks)

(iii) The length of the projection of $\underline{x}$ on $\underline{y}$ (2marks)

d) Let $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

(i) Is A symmetric? Give reason. (1mk)

(ii) Obtain Eigen value (3marks)

QUESTION TWO [20 MARKS]

Let $\underline{X} = (X_1, X_2, X_3)'$ follow a $N\left[\begin{bmatrix}-3\\1\\4\end{bmatrix}, \begin{bmatrix}1 & -2 & 0\\-2 & 5 & 0\\0 & 0 & 2\end{bmatrix}\right]$ distribution and define $Y = X_1 + X_2$.

a) Determine the mean and variance of Y. (5 marks)

b) The variance covariance matrix of $Y = (Y_1, Y_2)$ where $Y_1 = x_1 - x_2$ and $Y_2 = x_1 + x_3$ (5marks)

c) Show that the sample mean is an unbiased estimator of $\underline{\mu}$ and that the sample variance is biased estimator of matrix Σ (10marks)

QUESTION THREE [20 Marks]

(a) Let $\underline{x}$ be a trivariate random vector such that

$E(\underline{x}) = 0$ and $var(\underline{x}) = \begin{bmatrix}2.5 & 0 & 10\\0 & 1.5 & 0\\10 & 0 & 1\end{bmatrix}$.Find the expected value of the quadratic form

$Q = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_1)^2$ (4mks)

(b) Using the variance-covariance matrix in part (c) find

(i) The variance of $Y = x_1 - 2x_2 + x_3$ (3mks)

(ii) The variance covariance matrix of $Y = (Y_1, Y_2)$ where $Y_1 = x_1 + x_2$ and $Y_2 = x_1 + 2x_2 + x_3$ (3mks)

(c) Consider the following $n = 3$ observations on $p = 2$ variables

Variable 1: $y_{11} = 2, y_{21} = 3, y_{31} = 4$

Variable 1: $y_{12} = 1, y_{22} = 2, y_{32} = 4$

(i) Compute the sample means $\bar{y}_1$ and $\bar{y}_2$ and the sample variances S_{11} and S_{22} (6mks)

(ii) Compute the sample covariance S_{12} and the sample correlation coefficientr_{12} and interpret these quantities (4mks)

QUESTION FOUR (20MARKS)

(a) Let $\underline{y}$ be a p-variate random vector with mean vector $\underline{\mu}$ and variance covariance matrix Σ, show that $E(\underline{YY'}) = \Sigma + \underline{\mu\mu'}$, hence show that $E(Y'AY) = trace(A\Sigma) + \underline{\mu}'A\underline{\mu}$ where A is a symmetric matrix of constants. (8mks)

(b) Find the symmetric matrix A for a quadratic form $Q(Y_1, Y_2, Y_3) = 9Y_1^2 + 16Y_1Y_2 + Y_2^2 + 8Y_1Y_3 + 6Y_2Y_3 + 3Y_3^2$. Hence obtain the expected value of $Q(Y_1, Y_2, Y_3)$ and $E(Y'AY)$ given that $\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 4 & 2 \\ -1 & 2 & 9 \end{pmatrix}$ (12mks)

QUESTION FIVE [20 MARKS]

(a) Given the data matrix $Y = \begin{bmatrix} 1 & 9 & 10 \\ 4 & 12 & 16 \\ 2 & 10 & 12 \\ 5 & 8 & 13 \\ 3 & 11 & 14 \end{bmatrix}$

Define $Y_c = Y - 1\,\overline{Y}'$ as the mean corrected data matrix.

(i) Obtain the mean corrected data matrix (4mks)

(ii) Obtain the sample covariance matrix (4mks)

(iii) The generalized variance and hence verify that columns of mean corrected data matrix are linearly dependent. (3mks)

(iv) Specify a vector $V' = [v_1\ v_2 v_3]$ that establishes the linear dependence. (3mks)

(b) Let $\underline{x}$ be a random vector having the covariance matrix

$$\Sigma = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 25 & -1 \\ 1 & -1 & 9 \end{bmatrix}$$

Obtain the population correlation matrix (ρ) and $V^{\frac{1}{2}}$ (6mks)