(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 404**

COURSE TITLE: DIFFERENTIAL TOPOLOGY

DATE: 06/11/2020 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE: COMPULSORY (30 MARKS)

(a) Define the following terms in relation to differential topology.

(i)An n-dimensional Topological manifold (2mks)

(ii)A chart (2mks)

(iii)An Atlas (2mks)

(b) Show that the map $S^n = \{P \in R^{n+1} / |P| = 1\}$ is an $n - dimensional$ manifold. (5mks)

(c) Let M be a manifold and A a smooth atlas on M .Show that $D(A)$ is a smooth atlas. (6mks).

(d) Let $f: M \to N$ be a smooth map where M and N are $n - dimensional$ manfolds.When does $p \in M$ become a regular point. (3mks)

(e) Define a tangent space over a smooth $n - dimensional$ manifold.(4mks)

(f) Using illustration state the inverse function theorem . (3mks)

(g) Let (M, A) and (N, B) be smooth manifolds and $p \in M$ .When does the continous map $f: M \to N$ become smooth at p. (3mks)

***QUESTION* TWO (20 mks)**

(a)Show that if $f: (M, u) \to (N, v)$ and $g: (N, v) \to (P, w)$ are smooth then the composition $gof: (M, u) \to (P, w)$ is a smooth manifold also. (8mks)

(b)Let U be an open set in R^1 and $f: U \to R^1$ a continuously differentiable map.Let C be the set of critical points of f such that

$C = \{x \in U: f(x) = 0\}$. Then show that $f(c)$ has measure zero in R^1. (12mks)

QUESTION THREE (20 marks)

(a)Let $f: X \to N$ be a smooth map, where X is a smooth manifold with boundary and N a smooth manifold .Let S be a closed embedded submanifold of N .Show that the set of points $x \in X$ where f is transversal to S, is an open set of X. (8mks)

(b) (i)Using three examples define an immersion of a topological manifold. (6mks)

(ii)Using illustration define a submersion of a manifold. (6mks)

QUESTION FOUR (20 marks)

(a)Show that if $y \in Y$ is a regular value of $f: X \to Y$ then $f^{-1}(y)$ is a manifold of dimension $n - m$,since $dim(X) = n$ $\dim(Y) = m$. (8mks)

(b)State the sard's theorem and give five areas where the theorem is applicable. (8mks)

(c)State the regular value theorem. (4mks)

QUESTION FIVE (20 marks)

(a)Show that if $f: M \to N$ is a smooth map where M is $n + k - dimensional$ and N is $n - dimensional$ then if $q = f(p)$ is a regular value then $f^{-1}(q) \subseteq M$ is a $k - dimensional$ smooth submanifold. (10mks)

(b)Using illustrations describe the rank of a linear transformation (10mks)