(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2019/2020 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: STA 454

COURSE TITLE: LARGE SAMPLE THEORY

DATE: 12/11/2020 **TIME:** 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE(30 MARKS)

1. (a) Briefly state the importance of using limits in asymptotic theory. (2 mks)

   (b) Explain the meaning of the following expressions. Give an example in each case.

      i. $f_n \sim f$ if $f_n/f = 1$ (2 mks)

      ii. $f_n = O(g_n)$ (3 mks)

   (c) Show that two sequences c_n and d_n are equivalent iff their relative error tends to zero as $n \to \infty$. (3 mks)

   (d) The average entry score for new students admitted at a particular university is 60 with a standard deviation of 8. What is the probability that the mean entry score for a random sample of 50 of these freshmen will be anywhere from 58 to 64? (5 mks)

   (e) A symmetric tetrahedron is tossed 600 times. Find the lower bound for the probability of getting 40 to 60 fours (5 mks)

   (f) Let a sequence $c_n = n$ and $d_n = 2n$. Show that $e^{c_n} = o(e^{y_n})$ (5 mks)

   (g) If $X_1, X_2, ..., X_n$ is a sequence of random variables and if mean, μ_n and standard deviation, σ_n of X_n exists for all n and $\sigma_n \to -\infty$ as $n \to \infty$. Show that $x_n - \mu_n \xrightarrow{P} 0$ as $n \to \infty$ (5 mks)

QUESTION TWO (20 MARKS)

2. (a) Suppose $X_n \xrightarrow{a.s} X$, or $X_n \xrightarrow{p} X$, or $X_n \xrightarrow{d} X$. Show that for any continuous function $g(.)$, $g(X_n)$ converges to $g(X)$ almost surely, or in probability, or in distribution. (6 mks)

   (b) Show that if $F_n \to F$, then the convergence is uniform; $\sup_x |F_n| - F(x) \to 0$ as $n \to \infty$ (7 mks)

   (c) A dice is thrown 9000 times and a throw of 1 or 3 is observed 3240 times.

      i. Show that the dice cannot be regarded as an unbiased one (3 mks)

      ii. Find the limits between which the probability of a throw of 1 or 3 lies. (4 mks)

QUESTION THREE (20 MARKS)

3. (a) Let X denote the number of flaws in a 1 inch length of copper wire. The probability mass function of X is presented in the following table.

X	0	1	2	3
$P(X = x)$	0.01	0.12	0.48	0.39

   One hundred wires are sampled from this population. What is the probability that the average number of flaws per wire in this sample is less than 0.5? (6 mks)

   (b) Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli random variables with parameter p. Verify the law of large numbers. (4 mks)

   (c) Suppose $X_n \xrightarrow{d} X$, $Y_n \xrightarrow{p} Y$ and $Z_n \xrightarrow{p} Z$ for some constant y and z. Show that $Z_n X_n + Y_n \xrightarrow{d} zX + y$ (10 mks)