(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: MAT 110

COURSE TITLE: **BASIC CALCULUS**

DATE: 13/5/2021 **TIME:** 9:00 A.M - 11:00 A.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

(a) Define the following terms

i. Function (2 marks)

ii. Implicit function

(3 marks)

(b) Evaluate $\lim_{t\to\infty} \frac{x^3+1}{3tx^3-4x+5}$

(c) Using 1st principal of differentiation find the derivative of $y = 3x^3 - 2x^2 + 2x + 4$ (5 marks)

(d) Given $f(t) = t^2 + 1$ $g(t) = \frac{3}{t}$ and $h(t) = 2t$ determine the following composite functions

i. $h(g(t))$

ii. $g(h(t))$

iii. $g(f(t))$ (4 marks)

iv. $f(g(t))$ (3 marks)

(e) Find $\frac{dy}{dx}$ given$x = sin2xy + e^{2xy}$ (2 marks)

(f) Find $\frac{dy}{dx}$ given $y = t^3 + 1, x = t^3 - 1$ at $t = 5$

(g) Find the equation tangent and normal given $x(t) = t^2 + 1$ and$y(t) = \sqrt{1+t}$ at the point $t = 3$ (5 marks)

(h) Differentiate

i. $y = \frac{2sin3x}{3x^2}$ (6 marks)

ii. $y = (1 + x^2)^5 lnx^2$

QUESTION TWO (20 MARKS)

(a) Given the equation of the curve $y = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x + 4$, investigate the nature of the stationary points hence plot the graph (10 marks)

(b) Using 1st principal of differentiation find the derivative of $y = \sin x$ (5 marks)

(c) State the Rolle's theorem hence find the value of c satisfiying the conclusion of Rolle's theorem for $f(x) = x^3 + 2x^2 - x - 1$ on the interval $(-1,1)$ (5 marks)

QUESTION THREE (20 MARKS)

(a) A particle P moves along a straight line OX. At time $t = 0$ P is at the point O and t seconds later its displacement $S\ m$ is given by $S = t^3 - 6t^2 + 9t$

i. Write an expression for velocity and acceleration of P at t seconds

ii. Find when and where the particle will be at instantaneously at rest

iii. Find when and where the particle will be when $\frac{d^2s}{dt^2} = 0$ (6 marks)

(b) An object is moving vertically according to the equation $s = 100t - t^2$where t is time in seconds and S is the height of the object above the ground in feet

i. Find the velocity of the object when $t = 5$ seconds

ii. What is the time when the object starts to move downwards?

iii. How high does the object go (6 marks)

(c) Find y' and y'' of $y = \frac{sinx}{x^2}$ and hence prove that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 + 2)y = 0$ (8 marks)

QUESTION FOUR (20 MARKS)

(a) Define the following terms

i. Normal and Tangent line

ii. Maximum and minimum points (4 marks)

(b) Find the derivatives of the following functions

i. $\frac{d^4y}{dx^4}$ of $y = 2\sin 5x$

ii. $2x^3 - 4yx^2 = cosy$

iii. $y = e^{2t}lnt\ sin3t$

iv. $y = sinx^3$ (12 marks)

(c) Show that the slope of the tangent to the graph of the equation $sinxy = x^2cosy$ at $(2, \frac{\pi}{2})$ is $\frac{\pi}{4}$ (4 marks)

QUESTION FIVE (20 MARKS)

(a) Prove that $\lim_{\theta \to 0} \frac{cos\theta - 1}{\theta} = 0$ (5 marks)

(b) Find y' given that $ylnx = xe^y - 1$ (4 marks)

(c) If the radius r of a sphere is increasing at $2cm/s$. Find the rate at which the volume of the sphere is increasing when radius is $3cm$ (leave your answer in terms of π) (3marks)

(d) Investigate the stationary values of the function $y = x^3 - 3x^2 + 3x + 8$ hence sketch the curve (6 marks)

(e) Evaluate the limit of $\lim_{x \to 0} \frac{1 - cosx}{x^2}$ (2 marks)