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(KNOWLEDGE FOR DEVELOPMENT)

KIBABII UNIVERSITY

(KIBU)

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

END OF SEMESTER EXAMINATIONS

FIRST YEAR FIRST SEMESTER

FOR THE DEGREE IN

(INFORMATION TECHNOLOGY/ COMPUTER SCIENCE)

COURSE CODE: BIT 111/CSC 112

COURSE TITLE: DISCRETE STRUCTURES (FOR IT)

DATE: ~~08/04/2021~~ 14/05/2021 **TIME:** 2.00 P.M. – 4.00 P.M.

INSTRUCTIONS

ANSWER QUESTIONS ONE AND ANY OTHER TWO.

QUESTION ONE (COMPULSORY) [30 MARKS]

a. Differentiate between the following concepts as used in the study of discrete structures

i. Functions and relations **[2 marks]**

ii. Propositional logic and predicate logic **[2 marks]**

iii. Permutation and combination **[2 marks]**

b. Evaluate $C_{(8,3)} + 2 * P_{(7,3)}$ **[3 marks]**

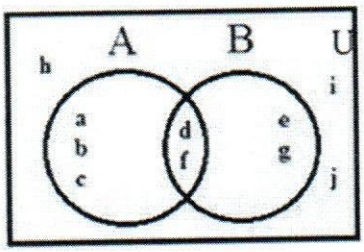
c. Check whether $((p \rightarrow q) \rightarrow (r \rightarrow s))$ and $((p \rightarrow r) \rightarrow (q \rightarrow s))$ are logically equivalent. **[3 marks]**

d. It is not the case that when **students are rowdy** or **the situation is violent then classes are cancelled**

i. Formalize the sentence in a logic form. **[2 marks]**

ii. Represent the statement on a truth table. **[4 marks]**

e. Study the Venn diagrams below and use it to find the following sets.



i. $(A \cup B)$ **[1 marks]**

ii. $A \cap B$ **[1 marks]**

iii. A' **[1 marks]**

iv. $|B - A|$ **[2 marks]**

v. $(A \cap B)'$ **[1 marks]**

vi. $(A \cup B)'$ **[1 marks]**

f. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$, $C = \{x, y, z\}$ and let $R = \{(1, a), (2, d), (3, a), (3, b), (3, d)\}$ and $S = \{(b, x), (b, z), (c, y), (d, z)\}$. Find $R \circ S$. **[4 marks]**

g. Find the inverse $g^{-1}(x)$ of $g(x) = \frac{11x^2 - 21}{7}$ **[3 marks]**

h. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea? **[3 marks]**

QUESTION TWO [20 MARKS]

a. Prove by the method of induction that for all n∈N then,

$$1.2.3+2.3.4+3.4.5+\ldots+n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$$ **[5 marks]**

b. In a survey of 80 people in Elegant Hotel 50 of them drink Tea, 40 of them drink Coffee and 20 drink both tea and coffee. Using the inclusive-exclusive principle find:

i. the number of people who take at least one of the two drinks **[2 marks]**

ii. Find the number of people who do not take any of the two stuff. **[2 marks]**

c. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of three popular options, air-conditioning (A), radio (R), and power windows (W), were already installed. The survey found: 15 had air-conditioning (A), 5 had A and W, 12 had radio (R), 9 had A and R, 3 had all three options. 11 had power windows (W), 4 had R and W. Using a Venn diagram, find the number of cars that had:

i. Only W; **[1 mark]**

ii. only A; **[1 mark]**

iii. only R; **[1 mark]**

iv. R and W but not A; **[1 marks]**

v. A and R but not W; **[1 marks]**

vi. only one of the options; **[2 marks]**

vii. at least one option; **[2 marks]**

viii. none of the options. **[2 marks]**

QUESTION THREE **[20 MARKS]**

a. "***Studying logic means studying proof***." Justify this claim. **[2 marks]**

b. Formalize the following sentence in predicate logic:

i. some person in this class smoke **[2 marks]**

ii. Everyone loves Mary **[2 marks]**

iii. Everyone loves someone **[2 marks]**

c. Construct the truth table of the following two formulas **(p ∧ ¬ (q ∨ r)) and (¬p ∨ (q ∨ r)).** Say for each one if it is a tautology, contingency, satisfiable, contradiction or logical consequence of the other. **[5 marks]**

d. Given a predicate P(x) such that $x^2+5>10$ and x consist of Positive integers ≤ 5. What is the truth value of:

$\forall x\ P(x)$ **[3 marks]**

$\exists x\ P(x)$ **[3 marks]**

QUESTION FOUR **[20 MARKS]**

a. Let R be the relation on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ defined by the rule $(a, b) \in R$ if the integer $(a - b)$ is divisible by 2.

i. Write R as a set of ordered pairs. **[3 marks]**

ii. Draw a directed graph of R. **[2 marks]**

iii. Write down the matrix of relation R. **[2 marks]**

iv. Find the inverse relation R^{-1} of R and describe it in words. **[2 marks]**

b. Let $f(x) = 4x^2$ be the function from R to R and let $g(x) = \sqrt{(x + 1)}$ be the function from $[0, \infty)$ to R. Find:

i. $g \circ f$ **[2 marks]**

ii. Domain of $g \circ f$ **[1 marks]**

iii. codomain (target) and range of $g \circ f$ **[2 marks]**

iv. $f \circ g(2)$ **[2 marks]**

c. Let $f: R \rightarrow R$ be defined by $(x) = 2x^2 + 2x - 12$. Justify whether f is

i. one-to-one; **[2 marks]**

ii. Onto. **[2 marks]**

QUESTION FIVE **[20 MARKS]**

a. A class contains 10 students with 6 men and 4 women. Find the number n of ways to:

i. Select a 4-member committee from the students. **[2 marks]**

ii. Select a 4-member committee with 2 men and 2 women. **[2 marks]**

iii. Elect a president, vice president, and treasurer. **[2 marks]**

b. Prove that: **[5 marks]**

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

c. Find the gcd(92928, 123552) hence or otherwise find the value of (s) and (**t**) in 92928(**t**)+123552(**s**)= gcd(92928, 123552). **[6 marks]**

d. Write the output of the following circuit at Q. **[3 marks]**