KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER EXAMINATIONS
FOR THE DEGREE
OF
BACHELOR OF SCIENCE.

FINAL EXAMINATIONS

COURSE CODE: **SPC 212**

COURSE TITLE: **VIBRATIONS AND WAVES**

DATE: 17/06/2021 **TIME:** 2-4PM

INSTRUCTIONS TO CANDIDATES

- Question ONE is compulsory and carries 30 marks
- Attempt any two of the remaining questions. Each carries 20 marks.
- Symbols used here hold their usual meaning Speed of sound in air $v = 343\text{m/s}$
- $\dot{x} = \frac{dx}{dt}$; $\ddot{x} = \frac{d^2x}{dt^2}$

KIBU observes ZERO tolerance to examination cheating

This paper consists of 4 printed pages. Please Turn *Over*

QUESTION ONE [30 Marks]

a) A block of mass m = 100 g attached to a horizontal spring with k = 0.4 N/m is in simple harmonic motion with a displacement given by x = - 0.2 cos(ωt). Calculate the period, T, and and the velocity of the block at time t = 3T/8. [4 marks]

b) Write down the differential equation that characterizes any simple harmonic oscillation, and its general solution. List the three conditions that must be satisfied for simple harmonic motion to occur in a mechanical system. [4 marks]

c) A particle of mass 100 g executes simple harmonic motion about x = 0 with angular frequency ω = 10 s^{-1}. Its total mechanical energy is E_{tot} = 0.45 J. Find the displacement of the particle when its speed is 2 m/s. [3 marks]

d) A damped oscillator is driven by a force $F(t) = F_0 \cos(\omega_e t)$. Explain briefly what is meant by the *transient* and the *steady-state* solutions for the displacement x(t), and write down the general form of the steady-state solution. [3 marks]

e) State the principle of linear superposition and give two examples of physical phenomena that rely on it. [3 marks]

f) Write down the general form for a harmonic wave travelling along the x axis, in terms of k, ω and ϕ_0. Determine the wavelength, frequency, phase constant, and phase speed of the wave

$$y(x,t) = 0.5\sin\left(0.2\pi x + 3\pi t + \frac{\pi}{6}\right),$$

in which the units of x and y are metres and t is in seconds. [3 marks]

g) Write down the one-dimensional wave equation, and show that the function $y(x,t) = 2e^{-3x+6t}$ is a solution
[3 marks]

h) A violin string must be tuned to vibrate at a frequency of 660 Hz in its fundamental mode. The vibrating part of the string is 33 cm long, and the linear density is 3 g/m. What is the tension when the string is in tune? [3 marks]

i) The equation of motion of a damped oscillator is

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \omega_0^{\ 2} x = 0$$

Briefly state what each term in this equation represents physically. [2 marks]

Consider the special case in which $\gamma = 2m\omega_0$

i. What is this type of damping, called? [1 mark] ii.

Re-write the equation of motion with ω_0 as the only constant. [1 mark]

QUESTION TWO [20 Marks]

A particle of mass m is in simple harmonic motion about an equilibrium position x = 0, with an angular frequency ω.

(a) Write down the differential equation of motion for the particle. Give the general solution for $x(t)$, and its first two derivatives, $\dot{x}(t)$ and $\ddot{x}(t)$. [4 marks]

(b) If the potential energy is defined to be U = 0 at equilibrium, show that $U = \frac{1}{2}m\omega^2 x^2$ in general. (Hint: use the work-energy theorem.) [4 marks]

(c) Use the results from (a) and (b) and the general definition of kinetic energy K, to find K and U as functions of time. Use these to find an expression for the total mechanical energy, Etot = K + U. [4 marks]

(d) Obtain a formula for the kinetic energy in terms of m, ω, x and the amplitude of oscillation. Find the displacement x at which K = U. [4 marks]

(e) Sketch the dependence of K, U, and Etot on displacement x. Label the minimum and maximum values of all quantities. [4 marks]

QUESTION THREE [20 Marks]

(a) A standing wave on a string for which the wave speed is v has the equation $y(x,t) = A\sin(kx)\cos(\omega t)$

If the string has fixed ends at x = 0 and x = L, then derive the allowed wavelengths, λ_n and Frequencies f_n of the normal modes. [8 Marks]

(b) If the wavefunction in (a) is re-written $y(x,t) = \psi(x)\cos(\omega t)$, then give the formula for $\psi_n(x)$ of the normal modes, in terms of n and L. [4 Marks]

(c) Sketch $\psi_n(x)$ for the first three harmonics, indicating clearly all nodes and antinodes. [4 Marks]

(d) Show that the normal modes satisfy the equation [4 Marks]

$$\frac{d^2\psi_n(x)}{dx^2} + \frac{n^2\pi^2}{L^2}\psi_n(x) = 0$$

QUESTION FOUR [20 Marks]

The equation of motion of a damped oscillator is given in equation (1)

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \omega_0^2 x = 0 \qquad (1)$$

This has three classes of solution given by equations (2), (3) and (4) below for the displacement x:

$$x(t) = e^{-\gamma t/2m}(C_1 t + C_2) \qquad (2)$$

where $q \equiv \sqrt{\frac{\gamma^2}{4m^2} - \omega_0^2}$ in equation (3), and $\omega \equiv \sqrt{\omega_0^2 - \frac{\gamma^2}{4m^2}}$ in equation

(a) A steel block of mass m = 8 kg is attached to a spring with $k = 64\,N\,m^{-1}$ and a damping constant, $\gamma = 48\,kg\,s^{-1}$. The block is in equilibrium at t = 0, when it receives an impulse that gives it an initial velocity of +2.5 ms^{-1}.

(i) Which of solutions given by equations (2), (3) or (4) above describes the motion of the block at $t \geq 0$? Justify your answer. [4 Marks]

(ii) Use the information given to determine the values of all constants in the appropriate x(t) equation, and thus specify both the displacement and the velocity of the block as functions of time. [6 Marks]

(b) Another block+spring system has the same m and k as in (a), but a different, γ, and is additionally subjected to a harmonic external force that varies in time with angular frequency ω_e and amplitude F_0

(i) (Write down the equation of motion for this system. [2 Marks]

(ii) Briefy explain what is meant by the *transient* and *steady-state* solutions in this case, and write down the general form of the steady-state solution. [4 Marks]

(iii) Calculate the maximum value of Y for which the driving force could possibly cause resonance. [4 Marks]