(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER EXAMINATIONS

FOR THE DEGREE OF
MASTER OF SCIENCE IN STATISTICS

COURSE CODE: STA 808

COURSE TITLE: TIME SERIES

DATE: 21/5/2021 **TIME:** 9:00 A.M – 12:00 NOON

INSTRUCTIONS TO CANDIDATES

- Answer question ONE (COMPULSORY) and any other TWO questions

QUESTION ONE (30 MARKS)

a) What is a time series?

b) For the AR(1) model, show that $\Gamma_0 = \dfrac{\sigma_\varepsilon^2}{(1-\phi_1^2)}$

c) Consider the ARMA (2, 2) model given by $X_t - X_{t-1} - \phi_1 X_{t-2} = \varepsilon_{t-2} + \varepsilon_{t-1} - 6\varepsilon_t$ where ϕ_1 is some constant and $\{\varepsilon_t\}$ is a sequence of shocks. Show that the process is invertible.

d) Obtain the Green function for the model

$$X_t - 1.5X_{t-1} + 0.6X_{t-2} = \varepsilon_{t-2} + \varepsilon_t - 0.5\varepsilon_t$$

Is the model stationary?

e) A series with 200 observations have a variance $\Gamma_0 = 10$. An AR (1) model with $\phi_1 = 0.8$ has been fitted to this series. Assuming the model to be adequate, find its variance σ_ε^2.

QUESTION TWO (20 MARKS)

The Green's function G_t of an ARMA (n, m) model is given by $G_t = 0.4(0.9)^{j-1}$ $j \geq 1$

Assume $X_0 = 0$ *and* ε_t*'s are*

t	0	1	2	3	4	5
ε_t	0	0.5	-1	1	-2	2

a) Compute X_5

b) Find the corresponding ARMA model and its parameters

QUESTION THREE (20 MARKS)

a) Derive an expression for the auto covariance function of an ARMA(1,1) model

b) Show that for an AR(2) model

$$\Gamma_0 = \frac{\sigma^2(1-\phi_2)}{(1+\phi_2)[(1-\phi_2)^2-\phi_1^2]}$$

$$\Gamma_1 = \frac{\phi_1}{(1-\phi_2)} \times Y_0$$

$$\Gamma_2 = \frac{(\phi_1^2+\phi_2-\phi_2^2)Y_0}{(1-\phi_2)}$$

QUESTION FOUR (20 MARKS)

a) Given the time series data $\{X_t, X_{t-1}, \ldots\}$. Suppose you are given that the ARMA (2,1) model if not adequate for this data. How will you discover this? Explain your answer. And if ARMA (2, 2) model is appropriate. How will you confirm this? Explain

b) Determine the asymptotic stability/stationary and stability of the following ARMA model

$X_t - 0.7X_{t-1} + 0.1X_{t-2} = \varepsilon_t - 1.7\varepsilon_{t-1} + 0.6\varepsilon_{t-2}$

c) Consider the process (X_t) given by

$X_t = X_{t-1} + \varepsilon_t$

i. Find the mean and variance of the process if $\varepsilon_t \sim N(0, \sigma_t^2)$

ii. Show that the process is stationary

QUESTION FIVE (20 MARKS)

a) Consider the process $(1-1.1B)(1-0.5B)X_t = (1-1.1B)\varepsilon_t$. Is the process invertible

b) Determine the asymptotic stability of the ARMA $(1-B^2)X_t = \varepsilon_t$

c) Obtain the Green's function for the process

$X_t - 0.5X_{t-1} = \varepsilon_t - 0.3\varepsilon_{t-1}$