(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

COURSE CODE: STA 112

COURSE TITLE: INTRODUCTION TO PROBABILITY

DATE: 21/5/2021 **TIME:** 9:00 A.M - 11:00 A.M

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

1. (a) Define the following terms: Sample outcome, Universal set, Mutually exclusive events (3 mks)

   (b) Let A be an event defined on a sample space S, show that $P(A) = 1 - P(A^c)$. (3 mks)

   (c) A woman has her purse snatched by two teenagers. She is subsequently shown a police lineup consisting of five suspects, including the two perpetrators. What is the sample space associated with the experiment "Woman picks two suspects out of lineup"? Which outcomes are in the event A: She makes at least one incorrect identification? (3 mks)

   (d) i. Explain the following: discrete random variable and continuous random variable (2 mks)

   ii. A random variable X has the probability distribution below

X	0	1	2	3	4	5	6	7	8
P(X=x)	a	2a	3a	4a	5a	6a	7a	8a	9a

   A. Determine the value of a (2 mks)

   B. Find $P(X < 3)$, $P(0 < X < 5)$ (4 mks)

   (e) A committee of 4 people need to be selected from 5 women and 7 men. How many ways can the committee be selected if at least 3 women must be included. (4 mks)

   (f) In a Bernoulli experiment,the first outcome has a probability x to occur, the second probability x^2. What is x? (3 mks)

   (g) A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3." Calculate $P(A|B)$; Are A and B independent? (3 mks)

   (h) Suppose $A \subset B$, show that $P(A) \leq P(B)$ (3 mks)

QUESTION TWO (20 MARKS)

2. (a) Let X be random variable with pdf

$$f(x) = \begin{cases} \frac{x}{5}, & x = 1, 2, 3, 4 \\ 0, & elsewhere \end{cases}$$

Compute;

i. $E(X)$, (2 mks)

ii. $E(3X)$ (3 mks)

iii. $Var(X)$ (3 mks)

(b) Suppose A and B be two events defined on a sample space S such that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \bigcup B) = 0.7$. Find;

i. $P(A \bigcap B)$ (2 mks)

ii. $P(A^c \bigcup B^c)$ (2 mks)

iii. $P(A^c \bigcap B)$ (2 mks)

(c) The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01. If a person tests positive, what is the probability that the person actually has the disease? Explain why a large proportion of those who test positive are actually disease free. (6 mks)

QUESTION THREE (20 MARKS)

3. (a) Three newspapers A, B and C are published in a town. It is estimated from survey that 20 percentage read A, 16 percentage read B and 14 percentage read C, 8 percentage read A and B, 5 percentage read A and C, 4 percentage read B and C, and 2 percentage read all the three papers. Represent this information on a Venn diagram and find the probability that a randomly chosen person:

i. does not read any paper (2 mks)

ii. reads A but not B (2 mks)

iii. does not read C (2 mks)

iv. reads only one of these papers (2 mks)

v. reads only two of these papers (2 mks)

(b) Let X have the pdf

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

Find

i. $Var(X)$ (5 mks)

ii. $Var(5X + 10)$ (5 mks)

QUESTION FOUR (20 MARKS)

4. (a) Consider tossing two fair dice. Let X denote the sum of the upturned values of the two dice and Y their absolute difference. Calculate the expected value of X and Y. (8 mks)

   (b) Let X (in tonnes) be a random variable representing the quantity of sugar sold in a day at a certain factory with a distribution function as shown;

$$f(x) = \begin{cases} Kx, & 0 \leq x \leq 3 \\ K(10 - x), & 3 < x \leq 5 \\ 0, & elsewhere \end{cases}$$

      i. Find K such that $f(x)$ is a pdf (4 mks)
      ii. Find $P(X \leq 3)$ (2 mks)
      iii. Find $P(X > 3)$ (2 mks)
      iv. Find $P(2.5 \leq X \leq 5)$ (4 mks)

QUESTION FIVE (20 MARKS)

5. (a) Two boxes each contain three cards. The first box contains cards labeled 1, 3 and 5. The second box contains cards labeled 2, 6, and 8. In a game, a player draws one card at random from each box and his score, X, is the sum of the numbers on the two cards.

      i. Obtain the six possible values of X and find their corresponding probabilities (2 mks)
      ii. Calculate the standard deviation of X. (8 mks)

   (b) A six-sided die has faces marked with the numbers 1, 3,5,7,9, and 11, it is biased so that the probability of obtaining the number R in a single roll of the die is proportional to R.

      i. Show that the probability distribution of R is given by $P(R = r) = \frac{r}{36}$, $r = 1, 3, 5, 7, 9, 11$ (3 mks)
      ii. The die is to be rolled and a rectangle drawn with sides of lengths 6cm and R cm. calculate the expected value of the area of the rectangle (4 mks)
      iii. The die is to be rolled again and a square drawn with sides of length $24R^{-1}$cm. calculate the expected value of the perimeter of the square (3 mks)