(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

MAIN CAMPUS

MASTERS FIRST YEAR SECOND SEMESTER EXAMINATIONS

COURSE CODE: STA 804

COURSE TITLE: STOCHASTIC PROCESS

DATE: 13/5/2021 **TIME:** 9:00 A.M – 12:00 NOON

INSTRUCTIONS TO CANDIDATES:
Answer Question one and any other two questions.

QUESTION ONE (30MARKS)

a) i. Define what is meant by probability generating function (pgf)

ii. If X is a random variable distributed as binomial with parameters n and P when n is known but P unknown. Obtain the pgf of X and hence compute $E[X^3 + 2X^2 + 3X + 1]$

b) For a Markov Chain. Show that $P(X_{n+3} = j \mid X_n = i) = P_{ij}^3$

c) In a branching process, the probability that any individual has j descendants is given by

$P_0 = 0$, $P_j = \frac{1}{2^j}; j \geq 1$. Show that $G(s) = \frac{s}{2-s}$. Hence compute

i. $G_2(S)$

ii. $G_3(S)$

d) In a 3-state Markov Chain with transition matrix

$$T_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

i. Show that E_1 is a recurrent non-null state

ii. Show that E_3 is a transient state

QUESTION TWO (20 MARKS)

Consider a birth –death process in which the probability of a birth and a death in a small time interval $(t, t+\Delta t)$ is $\lambda\Delta t$ and $\mu\Delta t$ respectively, where λ *and* μ birth and death rates respectively are

a) Obtain the difference-differential equation for the process

b) Calculate the pgf of the process

c) Compute the mean and variance of the process

QUESTION THREE (20 MARKS)

a) Discuss what is meant by branching process

b) Let X_n denote the population size at the n^{th} generation in a branching process. Show that

$V\text{ar}(X_n) = E[Var[X_n \mid X_{n-1}] + Var[E(X_n \mid X_{n-1})]$

 i. By Probability generating function technique

 ii. Conditional expectation.

c) If X is distributed as Poisson with parameter λ.

 i. Obtain the pgf of X

 ii. Hence compute the mean and variance

QUESTION FOUR (20 MARKS)

a) A simple random walk on a line or in one dimension occurs when a step forward has probability P and a step back has probability $q = 1 - P$. If X_n denotes the position of the walk at the n^{th} step.

 i. Obtain the mean of the walk

 ii. Obtain the variance of the walk

b) If $V_{n,x}$ denotes the probability that the walk is at state x after n-step. Show that

$$V_{n,x} = \binom{n}{\frac{1}{2}(n+x)}\frac{1}{2^n}$$

c) Find the probability that the event $X_6 = 4$ occurs in a random walk with P=0.3.

QUESTION FIVE (20 MARKS)

a) In a four-state Markov Chain has the transition matrix

$$T_4 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{3}{4} & 0 & \frac{1}{4} & 0 \end{pmatrix}$$

Find f_j the probability that the chain returns at some step to E_j for each state. Determine which states are transient and which states are recurrent. Which states forms a closed subset? Find the limiting behavior of T_4^n $as\, n \to \infty$.