(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE BACHELOR OF SCIENCE

COURSE CODE: **MAA 211 / MAT 203**

COURSE TITLE: **VECTOR ANALYSIS**

DATE: 16/06/2021 **TIME:** 9:00 AM-11:00 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE COMPULSORY (30 MARKS)

a) Define the following terms giving three examples for each.

  i. Scalar quantity

  ii. Vector quantity (4 marks)

b) Prove the associative law for vector addition. (4 marks)

c) Find $|\overrightarrow{AB}|$ if the position vectors of points A and B are $2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k}$. (3 marks)

d) If $\vec{A} = \sin xy\,\boldsymbol{i} + (3xy - 2x)\boldsymbol{j} + e^{xy}\,\boldsymbol{k}$, find

  i. $\frac{\partial \vec{A}}{\partial x}$ at (0,1)

  ii. $\frac{\partial \vec{A}}{\partial y}$ at (1,0) (5 marks)

e) If $\vec{A} = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$ and $\vec{B} = \boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$, find $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$ (5 marks)

f) Given $\vec{F} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$ *and* $\vec{r} = \boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$, find the magnitude of the torque or moment of the force $\vec{F}$. (3 marks)

g) If $|\vec{a}| = \mathbf{2}$, $|\vec{b}| = \mathbf{7}$ and $|\vec{a} \times \vec{b}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$. (2 marks)

h) Given $\vec{A} = \cos t\,\boldsymbol{i} + \sin 2t\,\boldsymbol{j} + e^{2t}\boldsymbol{k}$, determine

  i. $\frac{d}{dt}(\vec{A})$ (2 marks)

  ii. $\frac{d^2}{dt^2}(\vec{A})$ (2 marks)

QUESTION TWO (20 MARKS)

a) Determine a unit vector perpendicular to the plane that contains $\vec{A} = 2\boldsymbol{i} - 6\boldsymbol{j} - 3\boldsymbol{k}$ and $\vec{B} = 4\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ (3 marks)

b) Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{A} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\vec{B} = 3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ (3 marks)

c) Show that div curl $\vec{A} = 0$ where $\vec{A}$ is a vector field which has continuous second partial derivatives. (4 marks)

d) If the edges $\vec{a} = -3\boldsymbol{i} + 7\boldsymbol{j} + 5\boldsymbol{k}$, $\vec{b} = -5\boldsymbol{i} + 7\boldsymbol{j} - 3\boldsymbol{k}$ **and** $\vec{c} = 7\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$ meet at at a vertex, find the volume of the parallelopiped. (4 marks)

e) Prove that the points given by the vectors $4\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$, $\vec{b} = -\boldsymbol{i} - \boldsymbol{k}$, $3\boldsymbol{i} + 9\boldsymbol{j} + 4\boldsymbol{k}$ and $-4\boldsymbol{i} + 4\boldsymbol{j} + 4\boldsymbol{k}$ are coplanar. (6 marks)

QUESTION THREE (20 MARKS)

a) Define the terms

  i. divergence of a vector field

  ii. grad of a scalar field (4 marks)

b) Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$. (4 marks)

c) Given $\boldsymbol{A} = x^2y\boldsymbol{i} - 2xz\boldsymbol{j} + 2yz\boldsymbol{k}$, find

  i. div $\boldsymbol{A}$ (2 marks)

  ii. curl $\boldsymbol{A}$ (3 marks)

  iii. curl curl $\boldsymbol{A}$ (3 marks)

d) If $\emptyset = \frac{1}{\sqrt{x^2+y^2+z^2}}$ find gradient of $\emptyset$. (4 marks)

QUESTION FOUR (20 MARKS)

a) State without proving the Green's Theorem in the plane. (3 marks)

b) Show that;

  i. $\vec{V} = (x + 3y)i + (y - 2z)j + (x - 2z)k$ is a solenoidal vector field. (3 marks)

  ii. $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$ is a conservative vector field. (3 marks)

c) If $\vec{A}$ and $\vec{B}$ are differentiable vector functions of a scalar t, show that

$$\frac{d(\vec{A}\cdot\vec{B})}{dt} = \vec{A}\cdot\frac{d(\vec{B})}{dt} + \frac{d(\vec{A})}{dt}\cdot\vec{B}$$ (3 marks)

d) A particle moves along a curve whose parametric equations are $x = e^{-2t}$, $y = 2cost\ 3t$, $z = 2\sin 3t$ where t is time, determine its velocity and acceleration at $t = 0$. (5 marks)

e) Find the unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$. (3 marks)

QUESTION FIVE (20 MARKS)

a) Define line integral. (3 marks)

b) If $\vec{R}(t) = (t - t^2)\boldsymbol{i} + 2t^3\boldsymbol{j} - 3\boldsymbol{k}$, find $\int_0^1 \vec{\boldsymbol{R}}(\boldsymbol{t})\, \boldsymbol{dt}$ (5 marks)

c) If $\boldsymbol{F} = (3x^2 + 6y)\boldsymbol{i} - 14yz\boldsymbol{j} + 20xz^2\boldsymbol{k}$ evaluate $\int_C \boldsymbol{F}\cdot\boldsymbol{dr}$ from (0,0,0) to (1,1,1) along the paths C I the straight line from (0,0,0) to (1,0,0.), then to (1,1,0) and then to (1,1,1). (6 marks)

d) Evaluate $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ using the Green's theorem in the plane. (6 marks)